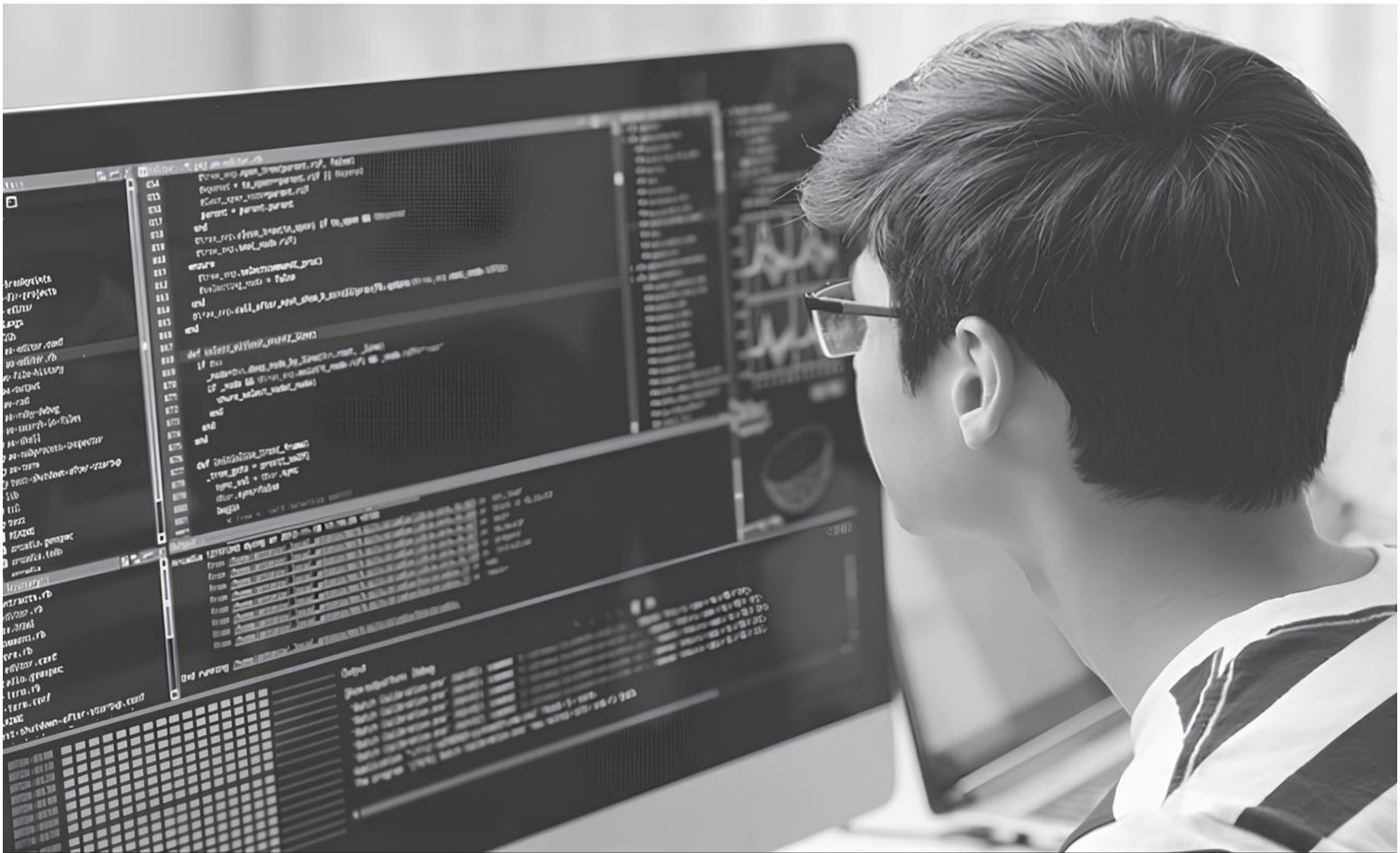
2

Relations and Functions



In computer programming languages like C, JAVA, etc., functions are essential. Functions are defined and reused while building computer programs, which really lessens the diffculty of repeatedly writing the complex codes again and again.

Topic Notes

- *Cartesian Products of Sets and Relations*
- *Functions and it's Types*

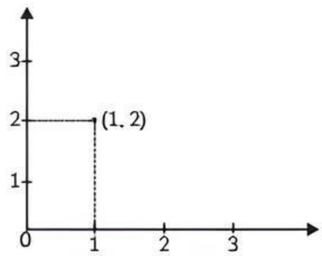
CARTESIAN PRODUCTS OF SETS AND RELATIONS 1

TOPIC 1

ORDERED PAIRS

The pair that is formed by two elements that are separated by a comma and written inside the parantheses.

Eg. (a, b) represents a ordered pair, where a is first element (or first component) and b is second element (or second component).



Important

➥ *Two ordered pairs are equal if their corresponding elements are equal.*
I.e. $(a, b) = (c, d) \Rightarrow a = c$ and $b = d$.

Graphically, ordered pair (a, b) represents a point in cartesian plane.

So, ordered pair (1, 2) implies that abcissa $x = 1$ and ordinate $y = 2$.

Example 1.1: Find the values of a and b, if

(A) $(2a - 4, 3) = (4, b + 6)$

(B) $(a - 5, b + 9) = (5, 9)$

Ans. (A) $2a - 4 = 4$ and $3 = b + 6$

$2a = 8$ and $b = 3 - 6$

$a = 4$ and $b = -3$

(B) $a - 5 = 5$ and $b + 9 = 9$

$a = 10$ and $b = 0$

TOPIC 2

CARTESIAN PRODUCTS OF SETS

Introduction

Relations and Functions give us the link between any two parameters. In our daily lives, we come across many patterns and links that characterise relations, such as a relation between a father and a son, brother and sister, etc.

A person owns one dog, and the dog is owned by one person. In relationships, a person has a partner, who is only partnered with that person. A person owns a car, and the car is owned by the person.

Cartesian Products of Two Sets

If A and B are two non-empty sets, then we define the Cartesian product $a \times b$ of sets A and B as

$$A \times B = \{|(a, b)\ |a \in A \text{ and } b \in B\}.$$

If $A = \phi$ or $B = \phi$, then we define the Cartesian product $A \times B$ of sets A and B as $A \times B = \phi$.

Cartesian Products of Three Sets

If A, B, and C are three non-empty sets, then we define the cartesian product $a \times b \times c$ of sets A, B, and C as $A \times B \times C = \{(a, b, c) : a \in A, b \in B, \text{ and } c \in C\}$.

If $A = \phi$ or $B = \phi$ or $C = \phi$, then we define cartesian product $A \times B \times C$ of sets A, B, and C as, $A \times B \times C = \phi$.

The element (a, b, c) is called an ordered triplet.

Example 1.2: If $G = \{7, 8\}$ and $H = \{5, 4, 2\}$, find $G \times H$ and $H \times G$. Are these products equal? **[NCERT]**

Ans. Given $G = \{7, 8\}$ and $H = \{5, 4, 2\}$.

Then, $G \times H = \{7, 8\} \times \{5, 4, 2\}$

$= \{(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)\}$

Now, $H \times G = \{5, 4, 2\} \times \{7, 8\}$

$= \{(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)\}$

Since, $G \times H$ and $H \times G$ do not have exactly the same ordered pairs.

$\therefore$ $G \times H \neq H \times G$.

Example 1.3: If $A = \{-1, 1\}$ then find $A \times A \times A$. **[NCERT]**

Ans. Given, $A = \{-1, 1\}$

$\therefore$ $A \times A \times A = \{-1, 1\} \times \{-1, 1\} \times \{-1, 1\}$

$A \times A \times A = \{(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1), (1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)\}$

Example 1.4: If set A has two elements and set $B = \{0, -1, -2\}$, then find the number of elements in $A \times B$. [NCERT]

Ans. It is given that, set A has 2 elements and the elements of set B are 0, –1 and –2.

No. of elements in $A = 2$

No. of elements in $B = 3$

So, no. of elements in $A \times B$

= No. of elements in A × No. of elements in B

$= 2 \times 3 = 6$

Thus, the number of elements in $(A \times B)$ is 6.

Example 1.5: Let $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$. Verify that

(A) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(B) $A \times C$ is a subset of $B \times D$ [NCERT]

Ans. (A) To verify: $A \times (B \cap C) = (A \times B) \cap (A \times C)$

We have,

$A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$

$\therefore$ L.H.S. $= A \times (B \cap C)$

$B \cap C = \{1, 2, 3, 4\} \cap \{5, 6\} = \phi$

Thus,

$A \times (B \cap C) = \{1, 2\} \times \phi = \phi$

$\therefore$ R.H.S. $= (A \times B) \cap (A \times C)$

$A \times B = \{1, 2\} \times \{1, 2, 3, 4\}$

$= \{(1, 1), (1, 2), (1, 3) (1, 4) (2, 1) (2, 2), (2, 3), (2, 4)\}$

$A \times C = \{1, 2\} \times \{5, 6\}$

$= \{(1, 5), (1, 6), (2, 5), (2, 6)\}$

$(A \times B) \cap (A \times C) = \phi$

Therefore, L. H. S. = R. H. S.

Hence, verified.

(B) To verify: $A \times C$ is a subset of $B \times D$.

$\therefore$ $A \times C = \{1, 2\} \times \{5, 6\}$

$= \{(1, 5), (1, 6), (2, 5), (2, 6)\}$

$\therefore$ $B \times D = \{1, 2, 3, 4\} \times \{5, 6, 7, 8\}$

$= \{(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)\}$

Since, all the elements of set $A \times C$ are the elements of set $B \times D$.

$\therefore$ $A \times C$ is a subset of $B \times D$.

Example 1.6: Let A and B be two sets such that $n(A) = 3$ and $n(B) = 2$. If $(x, 1)$, $(y, 2)$, $(z, 1)$ are in $A \times B$, find A and B, where x, y and z are distinct elements. [NCERT]

Ans. Given, $n(A) = 3$ and $n(B) = 2$; and $(x, 1)$, $(y, 2)$, $(z, 1)$ are in $A \times B$.

We know that,

A = Set of first elements of the ordered pair elements of $A \times B$.

B = Set of second elements of the ordered pair elements of $A \times B$.

So, clearly x, y, and z are the elements of A; and 1 and 2 are the elements of B.

As, $n(A) = 3$ and $n(B) = 2$, it is clear that set $A = \{x, y, z\}$ and set $B = \{1, 2\}$.

TOPIC 3

RELATION

A Relation R from a non-empty set A to a non-empty set B is a subset of the cartesian product set $A \times B$.

A Relation R between two sets A and B is defined as the subset of the cartesian product of A and B.

$$R \subset A \times B$$

Domain, Co-domain and Range of a Relation

Let R be a relation from a non-empty set A to a non-empty set B. Then:

(1) The set of all first elements of the ordered pairs in the relation R, is called domain of the relation R.

(2) The set of all second elements of the ordered pairs in the relation R, is called the range of the relation R.

(3) The set B is called the co-domain of the relation R.

For example, consider the relation R defined from $A = \{2, 3\}$ to $B = \{6, 9, 12\}$ as $R = \{(x, y) : x \in A, y \in B, y = 3x\} = \{(2, 6), (3, 9)\}$.

Then, the domain of $R = (2, 3\}$, co-domain of $R = \{6, 9, 12\}$, and the range of $R = \{6, 9\}$.

Important

- *The Domain of R is a subset of A.*
- *Range of R is a subset of B.*
- *Co-domain of R is B.*
  i.e. The range is always a subset of the co-domain.
- *If $n(A) = m$, $n(B) = n$; then the $n(A \times B) = mn$ and the total number of possible relations from the set A to set $B = 2^{m \times n}$.*

Representation of a Relation

A relation can be represented algebraically in roster form or in set-builder form and visually, it can be represented by an arrow diagram.

Roster Form

The relation is represented by the set of all ordered pairs belonging to R.

Set-Builder Form

The relation is represented by, R from set A to set B as $R = \{(a\ ,\ b\ ) : a \in A,\ b \in B$ and the rule which relate the elements of A and $B\}$.

Example 1.7: Write the given set in the roster form and set builder form. "Set of all two-digit numbers that are perfect square".

Ans. In the roster form:

$A = \{16, 25, 36, 49, 64, 81\}$

In the set builder form:

$A = \{x : x$ is a two digit perfect square number$\}$

Example 1.8: Write the given set in the roster form and set builder form. "Set of all natural numbers that can divide 24 completely."

Ans. In the roster form:

$A = \{1, 2, 3, 4, 6, 8, 12, 24\}$

In the set builder form:

$A = \{x : x$ is a natural number which divides 24 completely$\}$

Example 1.9: If $A = \{x, y, z\}$ and $B = \{1, 2\}$. Find the number of relations from A to B. [NCERT]

Ans. Here, the number of elements in $A = 3$

And, number of elements in $B = 2$

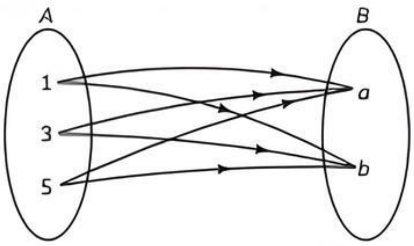
So, the number of relation from A to B is

$$= 2^{3\times2} = 2^6 = 64$$

Diagrammatic Representation of Cartesian Product of Two Sets

To represent $A \times B$ by an arrow diagram, we first draw Venn diagrams representing sets A and B, one opposite to the other, as shown in given figure. Now, we draw line segments starting from each element of set A and terminating with each element of set B.

If $A = \{1, 3, 5\}$ and $B = \{a, b\}$, then the following figure gives the arrow diagram of $A \times B$.



Example 1.10: Let $A = \{1, 2, 3, 4, 6\}$. Let R be the relation on A defined by $\{(a, b) : a, b \in A, b$ is exactly divisible by $a\}$.

(A) Write R in roster form.

(B) Find the domain of R.

(C) Find the range of R. [NCERT]

Ans. (A) Given, $A = \{1, 2, 3, 4, 6\}$

$a, b \in A$

Also, b is exactly divisible by a

Value of a	Value of b	$\frac{b}{a}$	Whether exactly divisible
1	1	$\frac{1}{1} = 1$	Yes
1	2	$\frac{2}{1} = 2$	Yes
1	3	$\frac{3}{1} = 3$	Yes
1	4	$\frac{4}{1} = 4$	Yes
1	6	$\frac{6}{1} = 6$	Yes
2	1	$\frac{1}{2}$	No
2	2	$\frac{2}{2} = 1$	Yes
2	3	$\frac{3}{2}$	No
2	4	$\frac{4}{2} = 2$	Yes
2	6	$\frac{6}{2} = 3$	Yes
3	1	$\frac{1}{3}$	No
3	2	$\frac{2}{3}$	No
3	3	$\frac{3}{3} = 1$	Yes
3	4	$\frac{4}{3}$	No
3	6	$\frac{6}{3} = 2$	Yes
4	1	$\frac{1}{4}$	No

Value of a	Value of b	$\frac{b}{a}$	Whether exactly divisible
4	2	$\frac{2}{4}$	No
4	3	$\frac{3}{4}$	No
4	4	$\frac{4}{4}=1$	Yes
4	6	$\frac{6}{4}$	No
6	1	$\frac{1}{6}$	No
6	2	$\frac{2}{6}$	No
6	3	$\frac{3}{6}$	No
6	4	$\frac{4}{6}$	No
6	6	$\frac{6}{6}=1$	Yes

$R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)\}$

(B) $R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)\}$

Domain of R = Set of first elements of relation R

$= \{1, 2, 3, 4, 6\}$

(C) $R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)\}$

Range of R = Set of second elements of relation

$= \{1, 2, 3, 4, 6\}$

Example 1.11: Let $A = \{1, 2, 3, \ldots 14\}$. Define a relation R from A to A by $R = \{(x, y) : 3x - y = 0$, where $x, y \in A\}$. Write down its domain, co-domain and range. [NCERT]

Ans. The relation R from A to A is given as,

$R = \{(x, y) : 3x - y = 0; x, y \in A\}$

i.e., $R = \{(x, y) : 3x = y; x, y \in A\}$

$\therefore$ $R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$

Value of x	Value of $y = 3x$	Whether $x, y \in A$
1	$3 \times 1 = 3$	Yes
2	$3 \times 2 = 6$	Yes
3	$3 \times 3 = 9$	Yes
4	$3 \times 4 = 12$	Yes
5	$3 \times 5 = 15$	No
6	$3 \times 6 = 18$	No
7	$3 \times 7 = 21$	No

Finding relation R

First, we check which values of x, y are in set A.

If both x and y are in set A, then $(x, y) \in R$

$\therefore$ $R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$

Domain of R = Set of all first elements of the ordered pairs

$= \{(1, 2, 3, 4\}$

Range of R = Set of all second elements of the ordered pairs

$= \{(3, 6, 9, 12\}$

Co-domain

R is defined from A to A.

$\therefore$ Co-domain of $R = A$

$= \{1, 2, 3, \ldots, 14\}$

Example 1.12: **Case Based:**

Ordered pair : The ordered pair of two elements a and b is denoted by (a, b) : a is first element (or first component) and b is second element (or second component).

Two ordered pairs are equal if their corresponding elements are equal.

i.e., (a, b) = (c, d) = a = c and b = d.

Cartesian product of two sets for two non-empty sets A and B, the Cartesian product A × B is the set of all ordered pairs of elements from sets A and B.

In symbolic form, it can be written as:

$$A \times B = \{(a, b) : a \in A, b \in B\}.$$

Based on the above topics, answer the following questions.

(A) If $(a - 3, b + 7) = (3, 7)$ then find the value of a and b.

(B) If $(x + 6, y - 2) = (0, 6)$, then find the value of x and y.

(C) Assertion (A): If $(x + 2, 4) = (5, 2x + y)$, then the value of x and y are $(-3, -2)$.

Reason (R): The ordered pairs are equal if their coresponding elements are equal.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

(D) Let A and B be two sets such that $A \times B$ are (1, 4), (2, 6) and (3, 6) then,

(a) $A \times B = B \times A$

(b) $A \times B \neq B \times A$

(c) $A \times B = \{(1, 4), (1, 6), (2, 4)\}$

(d) none of the above

(E) If $n(A \times B) = 45$, then $n(A)$ cannot be:

(a) 15 (b) 17

(c) 5 (d) 9

Ans. (A) We know that, two ordered paris are equal, if their corresponding elements are equal.

$$(a - 3, b + 7) = (3, 7)$$

$\Rightarrow$ $a - 3 = 3$

and $b + 7 = 7$

[equating corresponding elements]

$\Rightarrow$ $a = 3 + 3$ and $b = 7 - 7$

$\Rightarrow$ $a = 6$ and $b = 0$

(B) $(x + 6, y - 2) = (0, 6)$

$x + 6 = 0$

$\Rightarrow$ $x = -6$

$y - 2 = 6$

$\Rightarrow$ $y = 6 + 2 = 8$

(C) *(d) (A) is false but (R) is true.*

Explanation:

$$(x + 2, 4) = (5, 2x + y)$$

$x + 2 = 5$

$\Rightarrow$ $5 - 2 = 3$

$4 = 2x + y$

$\Rightarrow$ $4 = 2 \times 3 + y$

$\Rightarrow$ $y = 4 - 6 = -2$

(D) *(b) $A \times B \neq B \times A$*

Explanation: Since, (1, 4), (2, 6) and (3, 6) are elements of $A \times B$, it follows that 1, 2, 3 are elements of A and 4, 6 are elements of B. It is given that $A \times B$ has 6 elements.

So, $A = \{1, 2, 3\}$ and $B = \{4, 6\}$

Hence, $A \times B = \{1, 2, 3\} \times \{4, 6\}$

$= \{(1, 4), (1, 6), (2, 4), (2, 6), (3, 4), (3, 6)\}$

and $B \times A = (4, 6) \times (1, 2, 3)$

$= (4, 1) (4, 2), (4, 3), (6, 1), (6, 2), (6, 3)$

(E) *(b) 17*

Explanation: We have, $n(A \times B) = 45$

$\Rightarrow$ $n(A) \times n(B) = 45$

$\Rightarrow$ $n(A)$ and $n(B)$ are factors of 45 such that their product is 45.

Here, $n(A)$ cannot be 17.

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. If $A = \{1, 2, 4\}$, $B = \{2, 4, 5\}$ and $C = \{2, 5\}$, then $(A - B) \times (B - C)$ is:

(a) $\{(1, 2), (1, 5), (2, 5)\}$

(b) $\{(1, 4)\}$

(c) (1, 4)

(d) none of these

Ans. *(b) $\{(1, 4)\}$*

Explanation: Given,

$A = \{1, 2, 4\}$, $B = \{2, 4, 5\}$ and $C = \{2, 5\}$

$\therefore A - B = \{1, 2, 4\} - \{2, 4, 5\} = \{1\}$

$\therefore B - C = \{2, 4, 5\} - \{2, 5\} = \{4\}$

So, $(A - B) \times (B - C) = \{(1, 4)\}$

2. If R is a relation on the set $A = \{1, 2, 3, 4, 6, 7, 8, 9, 11, 12\}$ given by $x\,R\,y \Leftrightarrow y = 2x$, then R is equal to:

(a) $\{(2, 1), (4, 2), (8, 2), (9, 3)\}$

(b) $\{(2, 1), (4, 2), (6, 3)\}$

(c) $(5, 1), (2, 4), (3, 6)\}$

(d) none of these

Ans. *(b) $\{(2, 1), (4, 2), (6, 3)\}$*

Explanation: Given, R is a relation on the set $A = \{1, 2, 3, 4, 6, 7, 8, 9, 11, 12\}$ given by

$$x\,R\,y \Leftrightarrow y = 2x$$

Here, $y = 2x$

if, $x = 1, y = 2$

$x = 2, y = 4$

$x = 3, y = 6$

So, $R = \{(2, 1), (4, 2), (6, 3)\}$

3. The domain for which the functions defined by $f(x) = 3x^2 - 1$ and $g(x) = 3 + x$ are equal is:

(a) $\left\{-1, \frac{4}{3}\right\}$ (b) $\left\{-1, \frac{2}{3}\right\}$

(c) $\left\{-1, \frac{2}{3}\right\}$ (d) $\left\{-2, \frac{4}{2}\right\}$

[NCERT Exemplar]

Ans. *(a) $\left\{-1, \frac{4}{3}\right\}$*

Explanation: Given that

$f(x) = 3x^2 - 1$ and $g(x) = 3 + x$

$f(x) = g(x)$

$\Rightarrow$ $3x^2 - 1 = 3 + x$

$\Rightarrow$ $3x^2 - x - 4 = 0$

$\Rightarrow$ $3x^2 - 4x + 3x - 4 = 0$

$\Rightarrow$ $x(3x - 4) + 1(3x - 4) = 0$

$\Rightarrow$ $(x + 1)(3x - 4) = 0$

$\Rightarrow \quad x + 1 = 0$ or $3x - 4 = 0$

$\Rightarrow \quad x = -1,$ or $x = \frac{4}{3}$

$\therefore \quad \text{Domain} = \left\{-1, \frac{4}{3}\right\}$

4. **Let $S = \{x \mid x$ is a positive multiple of 3 less than 100$\}$, $P = \{x \mid x$ is a prime number less than 20$\}$. Then $n(S) + n(P)$ is:**

(a) 34 **(b) 41**

(c) 33 **(d) 30**

[Delhi Gov. SQP 2022]

Ans. *(b) 41*

Explanation: Given, $S = \{x \mid x$ is a positive multiple of 3 less than 100$\}$

So, $S = \{3, 6, 9, 12 \ldots\ldots, 99\}$

We can see that it is an A.P. with $a = 3$, $a_n = 99$, $d = 3$.

N^{th} term of an A.P. is given by:

$$a_n = a + (n - 1)d$$

$$\Rightarrow \quad 99 = 3 + (n - 1)3$$

$$\Rightarrow \quad \frac{96}{3} = n - 1$$

$$\Rightarrow \quad n = 33$$

Therefore, $n(S) = 33$

Given: $P = \{x \mid x$ is a prime number less than 20$\}$

So, $P = \{2, 3, 5, 7, 11, 13, 17, 19\}$

$n(P) = 8$

Now, $n(S) + n(P) = 33 + 8$

$\Rightarrow \quad n(S) + n(P) = 41$

5. **Let $A = \{a, b, c\}$, $B = \{1, 3, 5\}$. If relation R from A to B is given by $R = \{(a, 3), (b, 5), (c, 3)\}$. Then, R^{-1} is:**

(a) $\{(3, a), (5, b), (3, c)\}$

(b) $\{(1, a), (2, b), (3, c)\}$

(c) $\{(a, 3), (b, 2)\}$

(d) none of these

Ans. *(a) $\{(3, a), (5, b), (3, c)\}$*

Explanation: It is given that $R = \{(a, 3)\ (b, 5)\ (c, 3)\}$.

$\therefore \quad R^{-1} = \{(3, a), (5, b), (3, c)\}$

6. **$R = \{(x, y) \mid x, y \in N, x + 2y = 5\}$ is defined on set $A = \{1, 2, 3, 4, 5\}$ then the domain will be:**

(a) $\{1, 3\}$ **(b) $\{1, 2, 3\}$**

(c) $\{1, 2, 3, 4\}$ **(d) $\{1, 2, 3, 4, 5\}$**

[Diksha]

Ans. *(a) $\{1, 3\}$*

Explanation: Given that x, y are natural numbers and the relation is $x + 2y = 5$

Here, $x + 2y = 5$

For $x = 1$,

$1 + 2y = 5$

$2y = 4$

$y = 2$

For, $x = 2$, $2 + 2y = 5$

$2y = 3$

$y = \frac{3}{2} = 1.5$

If $x = 2$, then y becomes rational number which is not a natural.

For $x = 3$,

$3 + 2 \times y = 5$

$2y = 2$

$y = 1$

Thus, only 1 and 3 gives y as natural numbers 2 and 1.

Therefore, the domain is $\{1, 3\}$.

7. **If $A = \{1, 2, 3\}$, $B = \{1, 4, 6, 9\}$ and R is a relation from A to B defined by x is less than y. The range of R is:**

(a) $\{1, 2, 3\}$ **(b) $\{4, 6, 9\}$**

(c) $\{1, 3\}$ **(d) none of these**

Ans. *(b) $\{4, 6, 9\}$*

Explanation: The relation from A to B is x is less than y.

Hence, range is $\{4, 6, 9\}$

8. **Let f and g be two real functions given by**

$f = \{(0, 1), (2, 0), (3, -4), (4, 2), (5, 1)\}$

$g = \{(1, 0), (2, 2), (3, -1), (4, 4), (5, 3)\}$

then the domain of $f. g$ is:

(a) $\{2, 3, 4, 5, 6\}$ **(b) $\{1, 2, 3, 4\}$**

(c) $\{1, 2, 3, 4, 5\}$ **(d) $\{2, 3, 4, 5\}$**

[NCERT Exemplar]

Ans. *(d) $\{2, 3, 4, 5\}$*

Explanation: Given that:

$f(x) = \{(0, 1), (2, 0), (3, -4), (4, 2), (5, 1)\}$

and $g(x) = \{(1, 0), (2, 2), (3, -1), (4, 4), (5, 3)\}$

$\therefore$ domain of $f = \{0, 2, 3, 4, 5\}$

and domain of $g = \{1, 2, 3, 4, 5\}$

So, domain of $f. g$ = Domain of $f \cap$ Domain of g

$= \{2, 3, 4, 5\}$

9. **If $R = \{(x, y) : x, y \in Z, x^2 + y^2 \leq 16\}$ is a relation on Z, then the domain of R is:**

(a) $\{0, 1, 2, 3, 4\}$

(b) $\{0, -1, -2, -3, -4\}$

(c) $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

(d) none of these

Ans. *(c) $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$*

Explanation: Given, $R = \{(x, y) : x, y \in Z, x^2 + y^2 \leq 16\}$

Here, $x^2 + y^2 = 16$

$y^2 = 16 - x^2$

$y = \sqrt{16 - x^2}$

$16 - x^2 \geq 0$

$16 \geq x^2 - 4 \leq x \leq 4$

So, the domain is $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$.

10. A relation R is defined from {2, 3, 4, 5} to {3, 6, 7, 10} by : $x\ R\ y \Leftrightarrow x$ is factor of y. Then, the domain of R is:

(a) {2, 3, 5} (b) {3, 5}
(c) {2, 3, 4} (d) {2, 3, 4, 5}

[Delhi Gov. QB 2022]

Ans. *(a) {2, 3, 5}*

Explanation: Relations x is a factor of y. Here in set {2, 3, 4, 5} the factors of y are {2, 3, 5}.

So, domain of R = {2, 3, 5}.

11. How many elements will be there in the Cartesian product of A and B, if number of elements in A and B are respectively 10 and 7?

(a) 3 (b) 17
(c) 70 (d) 10^7 [Diksha]

Ans. *(c) 70*

Explanation: Given, $n(A) = 10$

$n(B) = 7$

So number of elements in $A \times B = n(A) \times n(B)$

$= 10 \times 7$

$= 70$

12. Let $n(A) = m$ and $n(B) = n$, then the total number of non-empty relations that can be defined from A to B is:

(a) m^n (b) $n^m - 1$
(c) $mn - 1$ (d) $2^{mn} - 1$

[NCERT Exemplar, Delhi Gov. SQP 2022]

Ans. *(d) $2^{mn} - 1$*

Explanation: Given that:

$n(A) = m$ and $n(B) = n$

$\therefore\ n(A \times B) = n(A).n(B) = mn$

So, the total number of relations from A to $B = 2^{mn} - 1$.

13. If A is the finite set and B is an infinite set, then what is $A \times B$?

(a) Infinite set (b) Finite set
(c) Undefined (d) A singleton set

Ans. *(a) Infinite set*

Explanation: If A is finite set, which means elements in A are countable. Set B has an infinite number of elements, which means set B contains uncountable elements. So, the cartesian product of A and B is infinite.

Assertion Reason Questions

Direction: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

14. Assertion (A): The domain of the relation $R = \{x + 2, x + 4 : x \in N, x < 8\}$ is {3, 4, 5, 6, 7, 8, 9}.

Reason (R): The range of the relation $R = \{(x + 2, x + 4 : x \in N, x < 8\}$ is {1, 2, 3, 4, 5, 6, 7}.

Ans. *(c) (A) is true but (R) is false.*

Explanation: The given relation is

$R = \{(3, 5) (4, 6) (5, 7) (6, 8) (7, 9) (8, 10) (9, 11)\}$.

Domain = {3, 4, 5, 6, 7, 8, 9}

Range of R = {5, 6, 7, 8, 9, 10, 11}.

15. Assertion (A): If $(x - 1, y + 2) = (2, 4)$, then $x = 3$ and $y = 2$.

Reason (R): Two ordered pairs (x, y) and (p, q) equal, if their corresponding elements are equal.

Ans. *(a) Both (A) and (R) are true and (R) is the correct explanation of (A).*

Explanation: Two ordered pairs are equal, then their corresponding elements are also equal.

So, $x - 1 = 2$

$\Rightarrow x = 3,$

$y + 2 = 4$

$\Rightarrow y = 2$

16. Assertion (A): If $(x, 1)$, $(y, 1)$ and $(z, 2)$ are in $A \times B$ and $n(A) = 3$, $n(B) = 2$, then $A = (x, y, z)$ and $B = \{1, 2\}$.

Reason (R): If $n(A) = 3$ and $n(B) = 2$. Then $n(A \times B) = 6$.

Ans. *(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).*

Explanation: It is given that $n(A) = 3$ and $n(B) = 2$ and $(x, 1), (y, 2), (z, 1)$ are in $A \times B$

We know that, A = set of first elements of the ordered pair elements of $A \times B$

B = Set of second elements of ordered pair elements of $A \times B$

$\therefore$ x, y and z are the elements of A and 1 and 2 are the elements of B.

Since, $n(A) = 3$ and $n(B) = 2$, it is clear that $A = \{x, y, z\}$ and $B = \{1, 2\}$

$n(A) = 3, n(B) = 2$

$\therefore n(A \times B) = n(A) \times n(B)$

$= 3 \times 2 = 6$

17. Assertion (A): Let $A = \{1, 2\}$ and $B = \{3, 4\}$ Then the number of relations from A to B is 16.

Reason (R): If $n(A) = p$ and $n(B) = q$, then number of a relation is 2^{pq}.

[NCERT Exemplar]

Ans. (a) *Both (A) and (R) are true and (R) is the correct explanation of (A).*

Explanation: Given, $A = \{1, 2\}$ and $B = \{3, 4\}$

No. of elements in set $A = n(A) = 2$

No. of elements in set $B = n(B) = 2$

No. of relations from A to $B = 2^{n(A) \times n(B)}$

$= 2^{2 \times 2} = 2^4 = 16$

18. Let $A = \{1, 2, 3, 4, 5, 6\}$. If R is the relation on A desired by $\{(a, b) : a, b \in A$ where b is square of $a\}$

Assertion (A): The relations R in roster form is $\{(1, 1), (2, 4), (3, 9), (4, 16) (5, 25), (6, 36)\}$.

Reason (R): The domain and range of R is $\{1, 2, 3, 4, 5, 6\}$.

Ans. (c) *(A) is true but (R) is false.*

Explanation: $A = \{1, 2, 3, 4, 5, 6\}$

$R = \{(a, b) : a, b \in A, b$ is exactly divisible by $a\}$

$R = \{(1, 1) (2, 4) (3, 9) (4, 16) (5, 25) (6, 36)(5, 25)\}$

Domain of $R = \{1, 2, 3, 4, 5, 6\}$

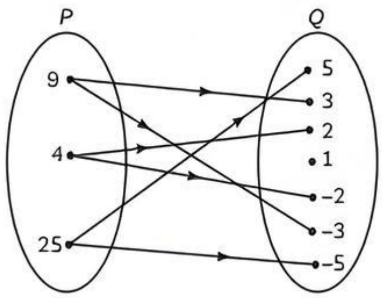
Range of $R = \{1, 4, 9, 16, 25, 36\}$

CASE BASED Questions (CBQs)

[**4** & **5** marks]

Read the following passages and answer the questions that follow:

19. *A class XI teacher, after teaching the topic of 'Relations', tries to assess the performance of her students over this topic. The figure shows a relation between the sets P and Q.*



(A) This relation in set builder form is:

(a) $R = \{(x, y) : x$ is square root of $y, x \in P, y \in Q\}$

(b) $R = \{(x, y) : y$ is square of $x, x \in P, y \in Q\}$

(c) $R = \{(x, y) : x$ is square of $y, x \in P, y \in Q\}$

(d) none of these

(B) The domain of relation is:

(a) $(1, 2, 3, 4, 5\}$ (b) $\{4, 9, 25, 5\}$

(c) $\{4, 9\}$ (d) $\{4, 9, 25\}$

(C) The range of relation is:

(a) $\{4, 9, 25\}$

(b) $\{1, 2, 3, 4, 5\}$

(c) $\{-2, 2, -3, 3, -5, 5\}$

(d) $\{-5, -3, -2, 1, 2, 3, 5\}$

(D) This relation in roster form is:

(a) $\{(9, 3), (4, 2), (25, 5)\}$

(b) $\{(9, 3), (9, -3), (4, 2), (4, -2), (25, 5), (25, -5)\}$

(c) $\{(9, -3), (4, -2), (25, -5)\}$

(d) none of the above

(E) The total number of relation from set P are:

(a) 32 (b) 64

(c) 128 (d) none of these

Ans. (A) (c) *$R = \{(x, y) : x$ is square of $y, x \in P, y \in Q\}$*

Explanation: Relation R is "x is the square of y".

$\therefore$ In set builder form, $R = \{(x, y) : x$ is the square of $y, x \in P, y \in Q\}$.

(B) (d) *{4, 9, 25}*

Explanation: The domain of relation is an element of set P i.e. $\{4, 9, 25\}$.

(C) (c) *{–2, 2, –3, 3, –5, 5}*

Explanation: The range of relation is $\{-2, 2, -3, 3, -5, 5\}$.

(D) (b) *{(9, 3), (9, –3), (4, 2), (4, –2), (25, 5), (25, –5)}*

Explanation: In roster form $R = \{(9, 3), (9, -3), (4, 2), (4, -2), (25, 5), (25, -5)\}$.

(E) *(b) 64*

Explanation: Total number of ordered pair in $R = 6$ (note that total no. of ordered pairs possible are $3 \times 7 = 21$)

$\therefore$ Total number of relation $= 2^6 = 64$

20. *Method to find the sets when cartesian product is given.*

For finding these two sets, we write the first element of each ordered pair in first set say A and corresponding second element in second set B (say).

Number of elements in cartesian product of two sets.

If there are p elements in set A and q elements in set B, then there will be pq elements in A × B i.e., if n (A) = p and n (B) = q, then n (A × B) = pq.

(A) If $A \times B = \{(a, 1), (b, 3), (a,3), (b, 1), (a, 2), (b,2)\}$. Then, find A and B. If the set A has 3 elements and set B has 4 elements, then find the number of elements in $A \times B$.

(B) The cartesian product $P \times P$ has 16 elements among which are found $(a, 1)$ and $(b, 2)$. Then find the set P.

(C) Express the function $f : A \text{—} R$, $f(x) = x^2 - 1$, where $A = \{-4, 0, 1, 4\}$ as a set of ordered pairs.

Ans. (A) Here, the first element of each ordered pair of $A \times B$ gives the elements of set A and the corresponding second element gives the elements of set B.

$\therefore$ $A = \{a, b\}$ and $B = \{1, 3, 2\}$

Given $n(A) = 3$ and $n(B) = 4$

$\therefore$ The number of elements in $A \times B$ is

$$n(A \times B) = n(A) \times n(B) = 3 \times 4 = 12$$

(B) Given, $n(P \times P) = 16$

$\Rightarrow$ $n(P).\ n(P) = 16$

$\Rightarrow$ $n(P) = 4$ ...(i)

Now, as $(a, 1) \in P \times P$

$\therefore$ $a \in P$ and $1 \in P$

Again, $(b, 2) \in P \times P$

$\therefore$ $b \in P$ and $2 \in P$

$\Rightarrow$ $a, b, 1, 2 \in P$

(C) Given, $A = \{-4, 0, 1, 4\}$

$f(x) = x^2 - 1$

$f(-4) = (-4)^2 - 1 = 16 - 1 = 15$

$f(0) = (0)^2 - 1 = -1$

$f(1) = (1)^2 - 1 = 0$

$f(4) = (4)^2 - 1 = 16 - 1 = 15$

Therefore, the set of ordered pairs = $\{(-4, 15), (0, -1), (1, 0), (4, 15)\}$

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

21. **Is $R = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$ a relation from set A to set B, where $A = \{1, 2, 3, 4\}$ and $B = \{1, 5, 9, 11, 15, 16\}$? Justify your answer.**

Ans. $A = \{1, 2, 3, 4\}$ and $B = \{1, 5, 9, 11, 15, 16\}$

$\therefore A \times B = \{(1, 1), (1, 5), (1, 9), (1, 11), (1, 15), (1, 16), (2, 1), (2, 5), (2, 9), (2, 11), (2, 15), (2, 16), (3, 1), (3, 5), (3, 9), (3, 11), (3, 15), (3, 16), (4, 1), (4, 5), (4, 9), (4, 11), (4, 15), (4, 16)\}$

It is given that $R = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$

A relation from a non-empty set A to a non-empty set B is a subset of the Cartesian product $A \times B$.

It is observed that R is a subset of $A \times B$

Thus R, is a relation from A to B.

22. **Let $A = \{a, b, c\}$, $B = \{b, c, d\}$ and $C = \{b, d\}$ then find $(A - B) \times (B - C)$.** **[Diksha]**

Ans. Given that,

$A = \{a, b, c\}$, $B = \{b, c, d\}$ and $C = \{b, d\}$

Now, $A - B = \{a\}$

And $B - C = \{c\}$

So, $(A - B) \times (B - C) = \{a\} \times \{c\} = \{(a, c)\}$

23. **Let $A = \{-1, -2\}$. Find the number of relations from A to A.**

Ans. Given, $A = \{-1, -2\}$

Number of elements $= n(A) = 2 = n$ (say).

Then, the number of relations from A to A

$$= 2^{n^2} = 2^{2^2} = 2^4 = 16$$

24. **If $P = \{x : x < 3, x \in N\}$, $Q = \{x : x \leq 2, x \in W\}$. Find $(P \cup Q) \times (P \cap Q)$, where W is the set of whole numbers.** **[NCERT Exemplar]**

Ans. Given that: $P = \{x : x < 3, x \in N\}$

$\Rightarrow$ $P = \{1, 2\}$

$Q = \{x : x \leq 2, x \in W\}$

$\Rightarrow$ $Q = \{0, 1, 2\}$

Now, $(P \cup Q) = \{0, 1, 2\}$ and $(P \cap Q) = \{1, 2\}$

$\therefore$ $(P \cup Q) \times (P \cap Q) = \{(0, 1), (0, 2), (1, 1), (1, 2), (2, 1), (2, 2)\}$

25. **Write the relation, $R = \{(x, x^4) : x$ is an even number less than 10$\}$ in roster form.**

Ans. Here, 2, 4, 6, 8 are the even numbers less than 10.

So, value of $x = 2, 4, 6, 8$

So, $(x, x^4) = (2, 16)\ (4, 256)\ (6, 1296)\ (8, 4096)\}$

So, $R = \{(2, 16)\ (4, 256)\ (6, 1296)\ (8,4096)\}$.

26. **Determine the domain and range of R.**

$R = \{(x, x + 5) : x \in 0, 1, 2, 3, 4, 5\}$

Ans. Given, $R = \{(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)\}$

Domain $= \{0, 1, 2, 3, 4, 5\}$

Range $= \{5, 6, 7, 8, 9, 10\}$

27. If $A = \{x : x \in W, x < 2\}$ $B = \{x : x \in N, 1 < x < 5\}$ $C = \{3, 5\}$ find:

(A) $A \times (B \cap C)$

(B) $A \times (B \cup C)$ [NCERT Exemplar]

Ans. Given that: $A = \{x : x \in W, x < 2\}$

$\Rightarrow$ $A = \{0, 1\}$

$B = \{x : x \in N, 1 < x < 5\}$

$\Rightarrow$ $B = \{2, 3, 4\}$

$C = \{3, 5\}$

(A) $A \times (B \cap C) = \{0, 1\} \times \{3\} = \{(0, 3), (1, 3)\}$

Hence, the cartesian product $= \{(0, 3), (1, 3)\}$

(B) $(B \cup C) = \{2, 3, 4\} \cup \{3, 5\}$

$= \{2, 3, 4, 5\}$

$A \times (B \cup C) = \{0, 1\} \times \{2, 3, 4, 5\}$

$= \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}$

28. The cartesian product $A \times A$ has 16 elements among which are found (–2, 1) (0, 3) and (1, 3). Find set A.

Ans. $(-2, 1) \in A \times A, (0, 3) \in A \times A$ and $(1, 3) \in A \times A$

Since $A \times A$ has 16 elements.

Hence,

$-2, 1, 0, 3 \in A$

So, A should have 4 elements. Only if A has elements $= \{-2, 1, 0, 3\}$.

29. If $\left(\frac{x}{4}+1, y-\frac{2}{4}\right) = \left(\frac{3}{4}, \frac{1}{4}\right)$, then find the values of x and y.

Ans. Given that $\left(\frac{x}{4}+1, y-\frac{2}{4}\right) = \left(\frac{3}{4}, \frac{1}{4}\right)$.

$\therefore$ Corresponding elements of above-ordered pairs must be equal.

We have,

$$\frac{x}{4}+1 = \frac{3}{4}$$

$$\frac{x}{4} = \frac{3}{4}-1$$

$$\frac{x}{4} = \frac{-1}{4}$$

$\Rightarrow$ $x = -1$

$$y-\frac{2}{4} = \frac{1}{4}$$

$$y = \frac{1}{4}+\frac{2}{4}$$

$\Rightarrow$ $y = \frac{3}{4}$

Hence, $x = -1, y = \frac{3}{4}$.

30. Given $R = \{(x, y) : x, y \in W, x^2 + y^2 = 64\}$, where W is the set of all whole numbers. Find the domain and range of R.

Ans. Here, $R = \{(x, y) : x, y \in W, x^2 + y^2 = 64\}$

So, $x = \sqrt{64 - y^2}$

So, $-8 \le y \le 8$

Hence, domain $= [-8, 8]$

And, range $= [0, 8]$

31. If $A = \{0, 1, 2\}$ and $B = \{3, 4\}$. Find domain of $A \times B$.

Ans. Given,

$A = \{0, 1, 2\}$ and $B = \{3, 4\}$

$A \times B = \{(0, 3)\ (0, 4)\ (1, 3)\ (1, 4)\ (2, 3)\ (2, 4)\}$

So, the domain of $A \times B = \{0, 1, 2\}$

Hence, domain of $A \times B = A$

32. Find the domain and range of the relation $R = \{(x, y) : y = x + \frac{8}{x}$, where $x, y \in N$ and $x < 8\}$.

Ans. Given, $R = \{(x, y) : y = x + \frac{8}{x}$, where $x, y \in N$ and $x < 8\} = \{(1, 9), (2, 6), (4, 6), (8, 9)\}$.

Hence, domain of $R = \{1, 2, 4, 8\}$ and range of $R = \{6, 9\}$.

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

33. Let $A = \{1, 3, 5, 7\}$ and $B = \{2, 4, 6, 8\}$ be two sets and let R be a relation from A to B defined by the phrase "$(x, y) \in R \Leftrightarrow x > y$".

Ans. Under this relation R, we obtain $3R2$, $5R2$, $5R4$, $7R4$ and $7R6$.

i.e. $R = \{(3, 2), (5, 2), (5, 4), (7, 2), (7, 6)\}$.

$\therefore$ Domain $(R) = \{3, 5, 7\}$ and range $(R) = \{2, 4, 6\}$.

34. In function $f = \{(1, 1), (0, -2), (3, 0), (2, 4)\}$ be a linear function defined by formula, $f(x) = ax + b$. Then find 'a' and 'b'. [Diksha]

Ans. Here, $f(x) = ax + b$

$f = \{(1, 1)\}\ (0, -2), (3, 0), (2, 4)\}$

and $f(x) = ax + b$

then, $f(0) = -2$

$-2 = a \times 0 + b$

$-2 = b$

and

$f(1) = 1$

$1 = a \times 1 + b$

$1 = a + b$

$1 = a - 2$

$3 = a$

35. If $f(x) = x^4$ then find the range of the function for $\{x = 1, 2, 3, 4, 5\}$.

Ans. Here $f(x) = x^4$

For $x = 1$

$f(x) = 1^4 = 1$

For $x = 2$

$f(x) = 2^4 = 16$

For $x = 3$

$f(x) = 3^4 = 81$

For $x = 4$

$f(x) = 4^4 = 256$

For $x = 5$

$f(x) = 5^4 = 625$

Hence, range of $f(x) = x^4$ for $x = 1, 2, 3, 4, 5$ is $= \{1, 16, 81, 256, 625\}$.

36. Define a relation R on the set N of natural numbers by $R = \{(x, y) ; y = x + 3, x$ is a prime number less than $8 : x, y \in N\}$. Depict this relationship using a roster form. Write down the domain and the range.

Ans. Given N = Set of all natural numbers and

$R = \{(x, y) : y = x + 3, x$ is a prime number less than $8 ; x, y \in N\}$

$= \{(x, y) : y = x + 3, x \in \{2, 3, 5, 7\} ; x, y \in N\}$.

The given relation in roster form can be written as

$R = \{(2, 5), (3, 6), (5, 8) (7, 10)\}$.

Hence, domain of $R = \{2, 3, 5, 8\}$ and range of $R = \{5, 6, 8, 10\}$.

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

37. In each of the following cases, find a and b.

(A) $(2a + b, a - b) = (8, 3)$

(B) $\left(\frac{a}{4}, a - 2b\right) = (0, 6 + b)$

[NCERT Exemplar]

Ans. (A) Given that: $(2a + b, a - b) = (8, 3)$

Comparing the domains and ranges, we get

$2a + b = 8$

$a - b = 3$

Now, $a = 3 + b$

Substituting the value of a in the equation $2a + b = 8$, we get

$2(3 + b) + b = 8$

$6 + 2b + b = 8$

$\Rightarrow 3b = 8 - 6 = 2$

$\Rightarrow b = \frac{2}{3}$

Substituting the value of b in eq. $(a - b = 3)$, we get

$a - \frac{2}{3} = 3$

$a = 3 + \frac{2}{3}$

$a = \frac{11}{3}$

(B) Given that: $\left(\frac{a}{4}, a - 2b\right) = (0, 6 + b)$

Comparing the domains and ranges, we get

$\frac{a}{4} = 0$

$\Rightarrow a = 0, a - 2b = 6 + b$

Now $\frac{a}{4} = 0$

$a = 0$

Substituting the value of a in eq.

$(a - 2b) = (6 + b)$

We get,

$0 - 2b = 6 + b$

$-2b - b = 6$

$-3b = 6$

$b = -2$

38. Let R be a relation from, Q and Q defined by $R = \{(p, q) : p, q \in Q$ and $q + p \in Z\}$. Show that

(A) $(p, p) \in R$, for all $p \in Q$.

(B) $(p, q) \in R$ implies that $(q, p) \in R$.

(C) $(p, q) \in R$ and $(q, r) \in R$ implies that $(p, r) \in R$.

Ans. Given, $R = \{(p, q) : p, q \in Q \text{ and } p - q \in Z\}$.

(A) Since, $p + p \in Z$, is true, where $p \in Q$.

$\Rightarrow \quad (p, p) \in R$.

(B) Let $(p, q) \in R$, where $p, q \in Q$.

$\Rightarrow \quad p + q \in Z$

$\Rightarrow \quad (p + q) \in Z$

$\Rightarrow \quad q + p \in Z$

$\Rightarrow \quad (q, p) \in R$.

(C) Let $(p, q) \in R$ and $(q, r) \in R$, where $p, q, r \in Z$

$\Rightarrow \quad p + q \in Z$ and $q + r \in Z$

$\Rightarrow \quad p + q = k_1$ and $q + r = k_2$, for some $k_1, k_2 \in Z$

On adding above equations, we get

$p + r = k_1 + k_2 \in Z$

$\Rightarrow \quad (p, r) \in R$.

39. **Let R be a relation N to N defined by $R = \{(a, b) : a, b \in N \text{ and } a = b^3\}$. Are the following true? Justify your answer in each case.**

(A) $(a, a) \in R$, for all $a \in N$

(B) $(a, b) \in R$ implies that $(b, a) \in R$

(C) $(a, b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$

Ans. Given, $R = \{(a, b) : a, b \in N \text{ and } a = b^3\}$.

(A) Since $a = a^3$, which is not true, for some $a \in N$

For $a = 2$, we have $(a, a) \notin R$

Therefore, $(a, a) \in R$ for all $a \in N$ is not true.

(B) Let $(a, b) \in R$, where $a, b \in N$

We know, $(8, 2) \in R$ since $8 = 2^3$

$\Rightarrow \quad a = b^3$.

$\Rightarrow \quad b \neq a^3$, for some $a, b \in N$

For $a = 8$, $b = 2$, we have $(a, b) \in R$ but $(b, a) \in R$.

(C) Let $(a, b) \in R$ and $(b, c) \in R$, where $a, b, c \in N$.

$\Rightarrow \quad a = b^3$ and $b = 3$

$\Rightarrow \quad a \neq c^3$, for some $a, c \in N$

For $a = 64$, $b = 4$, we have $(a, b) \in R$ and $(b, c) \in R$ but $(a, c) \notin R$.

40. **Let $f(x) = \sqrt{x}$ and $g(x) = x$ be two functions defined in the domain $R^+ \cup \{0\}$.**

(A) $(f + g)(x)$ **(B) $(f - g)(x)$**

(C) $(f \cdot g)(x)$ **(D) $\left(\frac{f}{g}\right)(x)$**

[NCERT Exemplar]

Ans. Given that: $f(x) = \sqrt{x}$ and $g(x) = x$ be two functions defined in the domain $R^+ \cup \{0\}$

(A) $(f + g)(x) = f(x) + g(x) = \sqrt{x} + x$

(B) $(f - g)(x) = f(x) - g(x) = \sqrt{x} - x$

(C) $(f \cdot g)(x) = f(x) . g(x) = \sqrt{x} . x = x^{3/2}$

(D) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}$

41. **Determine the domain and range of the relation R defined by**

(A) $R = \{(x, x + 6) : x \in \{0, 1, 2, 3, 4, 5\}$.

(B) $R = \{(x, x^4) : x$ is a prime number less than 20$\}$.

Ans. (A) $R = \{(x, x + 6) : x \in \{0, 1, 2, 3, 4, 5\}$

Given,

$R = \{(x, x + 6) : x \in \{0, 1, 2, 3, 4, 5\}$

$\therefore \quad R = \{(0, 0 + 6), (1, 1 + 6), (2, 2 + 6), (3, 3 + 6), (4, 4 + 6), (5, 5 + 6)\}$

$R = \{(0, 6), (1, 7), (2, 8), (3, 9), (4, 10), (5, 11)\}$

So,

Domain of relation, $R = \{0, 1, 2, 3, 4, 5\}$

Range of relation, $R = \{6, 7, 8, 9, 10, 11\}$

(B) $R = \{(x, x^4) : x$ is a prime number less than 20$\}$

Given,

$R = \{(x, x^4) : x$ is a prime number less than 20$\}$

Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19

$\therefore \quad R = \{(2, 2^4), (3, 3^4), (5, 5^4), (7, 7^4), (11, 11^4), (13, 13^4), (17, 17^4), (19, 19^4)\}$

$R = \{(2, 16), (3, 81), (5, 625), (7, 2401), (11, 14641), (13, 28561), (17, 83521), (19, 130321)\}$

So, Domain of relation

$R = \{2, 3, 5, 7, 11, 13, 17, 19\}$

Range of relation $R = \{16, 81, 625, 2401, 14641, 28561, 83521, 130321\}$

42. **Let a the relation R_1 on the set R of all real numbers be defined as $(a, b) \in R_1 \Leftrightarrow 1 + a b > 0$ for all $a, b \in R$.**

Show that:

(A) $(a, a) \in R_1$ for all $a \in R$

(B) $(a, b) \in R_1 \Rightarrow (b, a) \in R_1$ for all $a, b \in R$

Ans. (A) For any $a \in R$, we have

$1 + a^2 > 0 \Rightarrow (a, a) \in R_1$

Thus, $(a, a) \in R_1$ for all $a \in R$.

(B) Let $(a, b) \in R_1$. Then,

$(a, b) \in R_1 \Rightarrow 1 + a b > 0$

$\Rightarrow 1 + b a > 0 \Rightarrow (b, a) \in R_1$

Thus, $(a, b) \in R_1$

$\Rightarrow (b, a) \in R_1$ for all $a, b \in R$.

43. **Let $A = \{1, 2, 3, 4, 5 ...... 20\}$. Define a relation R from A to A by $R = \{(a, b) : a - 2b = 0, a, b \in A\}$. Depict the relation using roster form. Write domain and range of the relation.**

[Delhi Gov. SQP 2022]

Ans. The relation R from A to A is given as

$R = \{(a, b) : a - 2b = 0; a, b \in A)$

$R = \{(a, b) : a - 2b; a, b \in A\}$

$\therefore R = \{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7), (16, 8), (18, 9), (20, 10)\}$

The domain of R is the set of all first elements of the ordered pairs in the relation.

$\therefore$ Domain of $R = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$

The range of R is the set of all second elements of the ordered pairs in the relation

$\therefore$ Range of $R = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

44. **Let $A = \{1, 2, 3, 4, 5, 6\}$. Define a relation R from A to A by $R = \{(x, y) : y = x - 1\}$. Write R in roster form. Write down the domain, co-domain and range of R.**

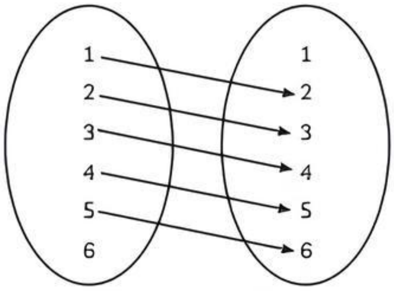
Ans. Given $A = \{1, 2, 3, 4, 5, 6\}$ and

$R = \{(x, y) : y = x - 1\}$.

The given relation in roster form can be written as,

$R = \{(1, 0), (2, 1), (3, 2), (4, 3), (5, 4), (6, 5)\}$

The given relation can be represented with the following arrow diagram:



So, domain of $R = \{1, 2, 3, 4, 5, 6\}$, co-domain of $R = \{0, 1, 2, 3, 4, 5\}$ and range of $R = \{2, 3, 4, 5, 6\}$.

45. **Is $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ a function? Justify: If this is described by the relation, $g(x) = \alpha x + \beta$, then what values should be assigned to α and β?** **[NCERT Exemplar]**

Ans. Given that $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$

Since every element of the domain in this relations has unique image.

So g is a function.

Now, $g(x) = \alpha x + \beta$

For (1, 1), $g(1) = \alpha(1) + \beta = 1$

$\Rightarrow \alpha + \beta = 1$ ...(i)

For (2, 3), $g(2) = \alpha(2) + \beta = 3$

$\Rightarrow 2\alpha + \beta = 3$ ...(ii)

Solving eqn. (i) and (ii) we get

$\alpha = 2$ and $\beta = -1$

$\therefore g(x) = 2x - 1$

[Note: We can take any other two ordered pairs]

Hence, the value of $\alpha = 2$ and $\beta = -1$.

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

46. **Let $A = \{-1, 2, 3\}$ and $B = \{1, 3\}$.**

Determine (A) $A \times B$ (B) $B \times A$ (C) $B \times B$ (D) $A \times A$

[NCERT Exemplar]

Ans. Given, $A = \{-1, 2, 3\}$ and $B = \{1, 3\}$

(A) $A \times B = \{-1, 2, 3\} \times \{1, 3\}$

So, $A \times B = \{(-1, 1), (-1, 3), (2, 1), (2, 3), (3, 1), (3, 3)\}$

Hence, the cartesian product are $\{(-1, 1), (-1, 3), (2, 1), (2, 3), (3, 1), (3, 3)\}$.

(B) $B \times A = \{1, 3\} \times \{-1, 2, 3\}$

Therefore, $B \times A = \{(1, -1), (1, 2), (1, 3), (3, -1), (3, 2), (3, 3)\}$.

Hence, the cartesian product are $\{(1, -1), (1, 2), (1, 3), (3, -1), (3, 2), (3, 3)\}$.

(C) $B \times B = \{1, 3\} \times \{1, 3\}$

So, $B \times B = \{(1, 1), (1, 3), (3, 1), (3, 3)\}$

Hence, the cartesian product are $\{(1, 1), (1, 3), (3, 1), (3, 3)\}$.

(D) $A \times A = \{-1, 2, 3\} \times \{-1, 2, 3\}$

So, $A \times A = \{(-1, -1), (-1, 2), (-1, 3), (2, -1), (2, 2), (2, 3), (3, -1), (3, 2), (3, 3)\}$

Hence, the cartesian product are

$\{(-1, -1), (-1, 2), (-1, 3), (2, -1), (2, 2), (2, 3), (3, -1), (3, 2), (3, 3)\}$.

47. **Let $A = \{-2, -1, 0, 1, 2\}$ and $f : A \to Z$ be given by $f(x) = x^2 - 2x - 3$. Find:**

(A) the range of f

(B) pre-images of 6, – 3 and 5.

Ans. (A) We have, $f(x) = x^2 - 2x - 3$,

$\therefore f(-2) = (-2)^2 - 2(-2) - 3 = 5$.

$f(-1) = (-1)^2 - 2(-1) - 3 = 0, f(0) = -3,$
$f(1) = 1^2 - 2 \times 1 - 3 = -4$ and
$f(2) = 2^2 - 2 \times 2 - 3 = -3.$

So, range $(f) = \{f(-2), f(-1), f(0), f(1), f(2)\}$
$= \{5, 0, -3, -4, -3\}.$

(B) Let x be a pre-image of 6. Then,

$f(x) = 6$

$\Rightarrow \quad x^2 - 2x - 3 = 6$

$\Rightarrow \quad x^2 - 2x - 9 = 0$

$\Rightarrow \quad x = 1 \pm \sqrt{10}$

Since, $x = 1 \pm \sqrt{10} \notin A.$

So, there is no pre-image of 6.

Let x be a pre-image of -3.

Then,

$f(x) = -3$

$\Rightarrow \quad x^2 - 2x - 3 = -3$

$\Rightarrow \quad x^2 - 2x = 0$

$\Rightarrow \quad x = 0, 2.$

Clearly, $0, 2 \in A.$

So, 0 and 2 are pre-images of -3.

Let x be a pre-image of 5. Then, $f(x) = 5$

$\Rightarrow \quad x^2 - 2x - 3 = 5$

$\Rightarrow \quad x^2 - 2x - 8 = 0$

$\Rightarrow \quad (x-4)(x+2) = 0$

$\Rightarrow \quad x = 4, -2.$

Since, $-2 \in A$ but $4 \notin A.$

So, -2 is a pre-image of 5.

48. Prove that:

(A) $(P \cup Q) \times R = (P \times R) \cup (Q \times R)$

(B) $(P \cap Q) \times R = (P \times R) \cap (Q \times R)$

Ans. (A) $(P \cup Q) \times R = (P \times R) = (P \times R) \cup (Q \times R)$

Let (x, y) be an arbitrary element of $(P \cup Q) \times R.$

$(x, y) \in (P \cup Q) \times R$

Since, (x, y) are elements of the cartesian product of $(P \cup Q) \times R.$

$x \in (P \cup Q)$ and $y \in R$

$(x \in P$ or $x \in Q)$ and $y \in R$

$(x \in P$ and $y \in R)$ or $(x \in Q$ and $y \in R)$

$(x, y) \in P \times R$ or $(x, y) \in Q \times R$

$(x, y) \in (P \times R) \cup (Q \times R)$ ...(i)

Let (x, y) be an arbitrary element of $(P \times R) \cup (Q \times R).$

$(x, y) \in (P \times R) \cup (Q \times R)$

$(x, y) \in (P \times R)$ or $(x, y) \in (Q \times R)$

$(x \in P$ and $y \in R)$ or $(x \in Q$ and $y \in R)$

$(x \in P$ or $x \in Q)$ and $y \in R$

$x \in (P \cup Q)$ and $y \in R$

$(x, y) \in (P \cup Q) \times R$ ...(ii)

From (i) and (ii), we get

$(P \cup Q) \times R = (P \times R) \cup (Q \times R)$

(B) $(P \cap Q) \times R = (P \times R) \cap (Q \times R)$

Let (x, y) be an arbitrary element of $(P \cap Q) \times R.$

$(x, y) \in (P \cap Q) \times R$

Since, (x, y) are elements of Cartesian product of $(P \cap Q) \times R$

$x \in (P \cap Q)$ and $y \in R$

$(x \in P$ and $x \in Q)$ and $y \in R$

$(x \in P$ and $y \in R)$ and $(x \in Q$ and $y \in R)$

$(x, y) \in P \times R$ and $(x, y) \in Q \times R$

$(x, y) \in (P \times R) \cap (Q \times R)$ ...(i)

Let (x, y) be an arbitrary element of $(P \times R) \cap (Q \times R).$

$(x, y) \in (P \times R) \cap (Q \times R)$

$(x, y) \in (P \times R)$ and $(x, y) \in (Q \times R)$

$(x \in P$ and $y \in R)$ and $(x \in Q$ and $y \in R)$

$(x \in P$ and $x \in Q)$ and $y \in R$

$x \in (P \cap Q)$ and $y \in R$

$(x, y) \in (P \cap Q) \times R$ ...(ii)

From (i) and (ii), we get:

$(P \cap Q) \times R = (P \times R) \cap (Q \times R)$

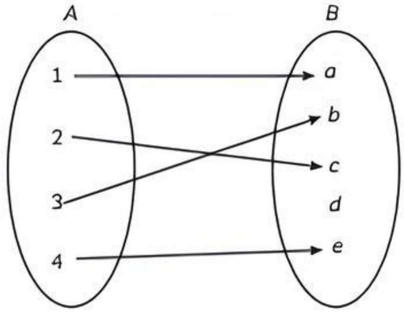
FUNCTIONS AND IT'S TYPES 2

TOPIC 1

FUNCTIONS

A function *f* from set *A* to *B* defines as a mapping between two sets that satisfy the following two conditions $f: A \to B$

1. Each every element of set *A* must be mapped with some elements of set *B*.
2. No elements of set *A* must be mapped with more than one element of set *B*.



Important

- *Function is a special type of relation.*
- *Every function is a relation, but every relation is not a function.*
- *If A has m elements and B has n elements, then the number of functions from A to B is n^m. and B to A is m^n.*

Domain and Co-Domain

Let $f: A \to B$, then set A is known as the domain of *f*, and set *B* is known as the Co-domain of *f*.

If $f: N \to R$ so, here *N* is the domain and *R* is the co-domain.

Range

The set of elements in set *B* that are mapped with elements of set *A*, is called the range of the function.

Range of $f \subseteq$ Co-domain of *f*.

Illustration: Let $A = \{-2, -1, 0, 1, 2\}$ and $B = \{0, 1, 2, 3, 4, 5, 6\}$. Consider a rule $f(x) = x^2$. Under this rule, we obtain $f(-2) = (-2)^2 = 4$, $f(-1) = (-1)^2 = 1$, $f(0) = 0^2 = 0$, $f(1) = 1^2 = 1$ and $f(2) = 2^2 = 4$. We observe that each element of *A* is associated to a unique element of *B*. So, $f: A \to B$ given by $f(x) = x^2$ is a function. Clearly, domain $(f) = A = \{-2, -1, 0, 1, 2\}$ and range $(f) = \{0, 1, 4\}$.

Equal Functions

The two functions *f* and *g* are said to be equal iff

1. domain of *f* = domain of *g*.
2. domain of *f* = co-domain of *g*.
3. $f(x) = g(x)$ for all *x* belonging to their common domain.

Illustration: Let $A = \{1, 2\}$, $B = \{3, 6\}$ and $f: A \to B$ given by $f(x) = x^2 + 2$ and $g: A \to B$ given by $g(x) = 3x$. Then, we observe that *f* and *g* have the same domain and co-domain. Also, we have, $f(1) = 3 = g(1)$ and $f(2) = 6 = g(2)$. Hence, $f = g$.

Example 2.1: Which of the following relations are functions? Give reasons. Also, determine its domain and range.

(A) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}

(B) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

(C) {(1, 3), (1, 5), (2, 5)}. [NCERT]

Ans. (A) Let $R = \{(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)\}$

Then, domain of $R = \{2, 5, 8, 11, 14, 17\}$ and range of $R = \{1\}$.

Since, every element in the domain has only one image.

Hence, *R* is a function.

(B) Let $R = \{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)\}$

Then, domain of $R = \{2, 4, 6, 8, 10, 12, 14\}$ and range of $R = \{1, 2, 3, 4, 5, 6, 7\}$.

Since, every element in the domain has only one image.

Hence, *R* is a function.

(C) Let $R = \{(1, 3), (1, 5), (2, 5)\}$

Then, domain of $R = \{1, 2\}$ and range of $R = \{3, 5\}$.

Since, element 1 in the domain does not have only one image. (1 has two images, 3 and 5).

Hence, *R* is not a function.

Example 2.2: Write total number of functions from set *A* to set *B*.

(A) $A = \{1, 2, 3\}, B = \{a, b, c\}$

(B) $A = \{1, 2, 3\}, B = \{a, b, c, d\}$ [NCERT]

Ans. (A) In set *A* no. of elements $m = 3$

In set *B* no. of elements $n = 3$

So, total no. of function from *A* to *B* is $= n^m$

$= 3^3 = 27$

(B) In set *A*, $m = 3$

In set *B*, $n = 4$

So total no. of function from *A* to *B* is $= n^m$

$= 4^3 = 64$

Example 2.3: Find the domain for which the function $f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$ are equal. **[NCERT]**

Ans. The values of x for which $f(x)$ and $g(x)$ are equal, are given by

$$f(x) = g(x)$$
$$\Rightarrow \quad 2x^2 - 1 = 1 - 3x$$
$$\Rightarrow \quad 2x^2 + 3x - 2 = 0$$
$$\Rightarrow \quad (x + 2)(2x - 1) = 0$$
$$\Rightarrow \quad x = -2, \frac{1}{2}$$

Thus, $f(x)$ and $g(x)$ are equal on the set $\left\{-2, \frac{1}{2}\right\}$.

Real-Valued Functions

A function $f: A \rightarrow B$ is called a real-valued function if B is a subset of R.

Real Function

A function $f: A \rightarrow B$ is called a real function if both A and B are subsets of R.

Domain of Real Function

The domain of $f(x)$ is the set of all those real numbers for which $f(x)$ is defined.

Range of Real Function

The range of a real function is the set of all real values taken by $f(x)$ at the points in its domain.

Steps to find Range

(i) Put $y = f(x) \Rightarrow x = g(y)$.

(ii) Find the value of x by solving the equation.

(iii) Find the values of y for which the values of x, obtain from $x = g(y)$, values of x are real, and in the domain of f.

(iv) The set of values of y obtained is the range of f.

Example 2.4: Find domain of $f(x) = \sqrt{x - 2}$. **[NCERT]**

Ans. Let,

$$y = f(x)$$
$$y = \sqrt{x-2}$$
$$x - 2 \geq 0$$
$$x \geq 2$$
$$x \in [2, \infty)$$

Hence, domain = $[2, \infty)$

Example 2.5: Find the range of $f(x) = \frac{x-2}{3-x}$. **[NCERT]**

Ans. Let,

$$y = f(x)$$
$$y = \frac{x-2}{3-x}.$$
$$3y - xy = x - 2$$
$$xy + x = 3y + 2$$
$$x(y + 1) = 3y + 2$$
$$x = \frac{3y+2}{y+1}$$

Here, x assumes real values at all points except when

$$y + 1 = 0 \text{ i.e. } y = -1$$

Hence, range $(f) = R - \{-1\}$.

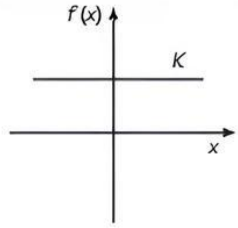
Some Functions and Their graphs

Constant Function

$$f(x) = k, \forall x \in R$$
Domain $= R$

Range of $f = \{k\}$

Graph of f is a line parallel to x-axis.

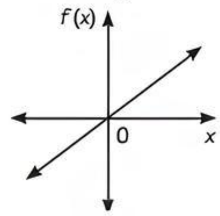


Identity Function

$$f(x) = x, \forall x \in R$$
Domain of $f = R$

Range of $f = R$

Graph of f is a straight line.

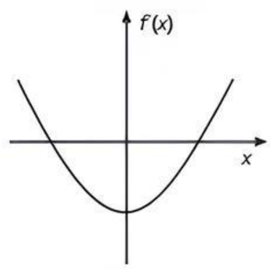


Polynomial Function

$$f(x) = a_0 + a_1 x + ... + a_n x^n \ \forall x \in R$$
Domain of $f = R$

Range of $f = R$

$$f(x) = x^2 + x - 1$$



Rational Function

$$f(x) = \frac{g(x)}{h(x)}, \forall x \in R \ h(x) \neq 0$$
Domain of $f \subseteq R$

Range of $f \subseteq R$

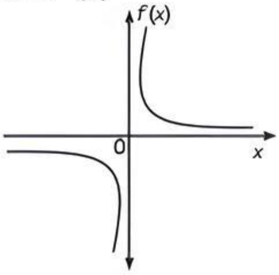
$$f(x) = \frac{x^2}{x+3}$$

Reciprocal Function

$f(x) = \frac{1}{x} \ \forall x \in R - \{0\}$

Domain of $f = R - \{0\}$

Range of $f = R - \{0\}$



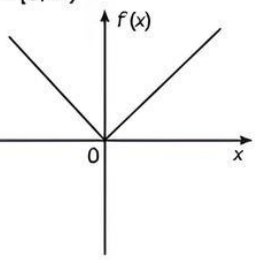
It is also a rational function.

Modulus Function

$f(x) = |x| \ \forall x \in R$

Domain of $f = R$

Range of $f = [0, \infty)$

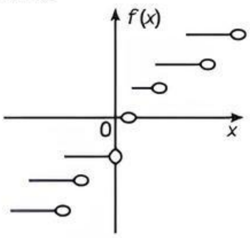


Greatest integer function

$f(x) = [x] \ \forall x \in R$

Domain of $f = R$

Range of $f = Z$

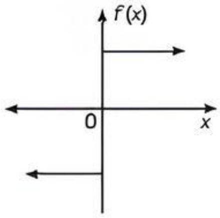


Signum function

$$f(x) = \begin{cases} 1 & x > 0 \\ 0 & > \; = 0 \ \forall x \in R \\ -1 & x < 0 \end{cases}$$

Domain of $f = R$

Range of $f = \{-1, 0, 1\}$

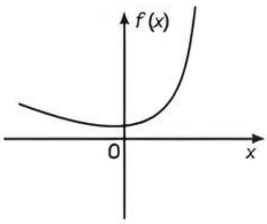


Exponential function

$f(x) = e^{x} \ \forall x \in R$

Domain of $f = R$

Range of $f = (0, \infty)$

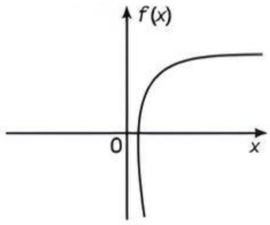


Logarithmic function

$f(x) = \log_b x \ \forall x \in (0, \infty)$

The domain of $f = (0, \infty)$

Range of $f = R$



Example 2.6: *A function f is defined by* $f(x) = 2x - 5$. Write values of:

(A) $f(0)$

(B) $f(7)$

(C) $f(-3)$ [NCERT]

Ans. $f(x) = 2x - 5$

So,

(A) $f(0) = 2 \times 0 - 5 = -5$

(B) $f(7) = 2 \times 7 - 5 = 9$

(C) $f(-3) = 2 \times -3 - 5 = -11$

Example 2.7: The function, 'F' which maps temperature in degree Celsius into temperature in degree Fahrenheit, is defined by $F(C) = \frac{9C}{5} + 32$.

(A) $F(10)$

(B) $F(15)$

(C) $F(-10)$

(D) The value of C, when $F(C) = 212$. [NCERT]

Ans. Given, $F(C) = \frac{9C}{5} + 32$

(A) $F(10) = \frac{9}{5}(10) + 32 = 50$

(B) $F(15) = \frac{9}{5}(15) + 32 = 59$

(C) $F(-10) = \frac{9}{5}(-10) + 32 = 14$

(D) We have,

$$F(C) = 212$$

$$\Rightarrow \quad \frac{9C}{5} + 32 = 212$$

$$\Rightarrow \quad \frac{9C}{5} = 180$$

$$\Rightarrow \quad C = 100$$

Example 2.8: If $f(x) = x^2$, then find $\frac{f(2.2) - f(2)}{2.2 - 2}$. [NCERT]

Ans. Given, $f(x) = x^2$

$$\therefore \quad \frac{f(2.2) - f(2)}{2.2 - 2} = \frac{(2.2)^2 - (2)^2}{(2.2) - (2)}$$

$$= \frac{(2.2 + 2)(2.2 - 2)}{(2.2 - 2)}$$

$$= 2.2 + 2 = 4.2$$

TOPIC 2

ALGEBRA OF REAL FUNCTIONS

Let $f : D \rightarrow R$ and $g : D \rightarrow R$ be any two real functions (where $D \subseteq R$). Then,

1. The sum of real functions f and g is a function.
   $f + g : D \rightarrow R$
   Defined by
   $(f + g)(x) = f(x) + g(x)$, for all $x \in D$.
2. The difference of real functions g from f is a function.
   $f - g : D \rightarrow R$
   Defined by
   $(f - g)(x) = f(x) - g(x)$, for all $x \in D$
3. The product (or pointwise multiplication) of real functions f and g is a function.
   $fg : D \rightarrow R$
   Defined by
   $(fg)(x) = f(x)\, g(x)$, for all $x \in D$.
4. Multiplication of real functions f by a scalar (i.e., a real number) α is a function $\alpha f : D \rightarrow R$.
   Defined by
   $(\alpha f)(x) = \alpha f(x)$, for all $x \in D$
5. The quotient of real functions f by g is a function.
   $\frac{f}{g} : D \rightarrow R$
   Defined by
   $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, for all $x \in D$ where $g(x) \neq 0$.

If the functions f and g have different domains, then we have the following conclusion:

Let f and g be two functions with domains D_1 and D_2 respectively. Let α be a scalar. Then,

1. The domain of $f + g = \{x : x \in D_1 \cap D_2\}$.
2. The domain of $f - g = \{x : x \in D_1 \cap D_2\}$
3. The domain of αf = Domain of f.
4. The domain of $fg = \{x : x \in D_1 \cap D_2\}$.
5. The domain of $\frac{f}{g} = \{x : x \in D_1 \cap D_2, g(x) \neq 0\}$.

Example 2.9: If $f(x) = x^3 + 1$ and $g(x) = x + 1$. Find $f + g$, $f - g$, fg and $\frac{f}{g}$. [NCERT]

Ans. $f + g = x^3 + 1 + x + 1 = x^3 + x + 2$

$f - g = x^3 + 1 - x - 1 = x^3 - x$

$f.g = (x^3 + 1)(x + 1) = x^4 + x^3 + x + 1$

$\frac{f}{g} = \frac{x^3 + 1}{x + 1}$

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. Let $A = \{a, b, c\}$, $B = \{2, 3, 4\}$, then which of the following is a function from A to B ?
   (a) $\{(a, 2), (b, 3), (c, 3), (c, 3)\}$
   (b) $\{(a, 3), (b, 4)\}$
   (c) $\{(a, 3), (b, 2), (c, 3)\}$
   (d) $\{(a, 2), (b, 3), (c, 2), (c, 4)\}$

Ans. (c) $\{(a, 3), (b, 2), (c, 3)\}$

Explanation: Given,

$A = \{a, b, c\}$, $B = \{2, 3, 4\}$

So, function from A to B is

$\{(a, 3)\ (b, 2)\ (c, 3)\}$

2. If $f: Q \to Q$ is defined as $f(x) = x^2$, then $f^{-1}(16)$ is:

(a) 4 (b) –4

(c) {–4,4} (d) ϕ

Ans. (c) {–4, 4}

Explanation: Given,

$$f(x) = x^2$$

Now, $y = x^2$

$$\sqrt{y} = x$$

i.e., $f^{-1}(x) = \sqrt{y}$

$$f^{-1}(16) = \sqrt{16} = \{-4, 4\}$$

3. If $p = e^{2x-2}$ then, log p at $x = 3$ is:

(a) 1 (b) 2

(c) 3 (d) 4

Ans. (d) 4

Explanation: Given, $p = e^{2x-2}$

Taking log both side

$$\log p = \log e^{2x-2}$$
$$= (2x-2)\log e$$
$$\log p = 2x - 2 \quad [\because \log e = 1]$$
$$\log p = 2 \times 3 - 2$$
$$= 6 - 2$$
$$= 4$$
$$\log p = 4$$

4. Let $f(x) = |x - 2|$. Then,

(a) $f(x^2) = [f(x)]^2$ (b) $f(x + y) = f(x) f(y)$

(c) $f(|x|) = |f(x)|$ (d) none of these

Ans. (d) none of these

Explanation: Given,

$$f(x) = |x - 2|$$

Here,

$$f(x^2) \neq [f(x)]^2$$

So, it is not true.

$$f(x + y) \neq f(x).f(y)$$

and

$$f(|x|) \neq |f(x)|$$

5. Domain of $\sqrt{a^2 - x^2}\,(a > 0)$ is:

(a) (–a, a) (b) [–a, a]

(c) (0, a) (b) (–a, 0]

[Delhi Gov. SQP 2022]

Ans. (b) [–a, a]

Explanation: Let $f(x) = \sqrt{a^2 - x^2}$

$f(x)$ of define only when $a^2 - x^2 \geq 0$

$$(a + x)(a - x) \geq 0$$
$$-(a + x)(x - a) \geq 0$$
$$(x - a)(x + a) \leq 0$$

Hence, $x \in [-a, a]\ \forall\ a > 0$

Domain = [–a, a]

6. If $f(x) = 2x^3 - \left(\frac{1}{x}\right)^{1/2}$, then $f(4)$ is:

(a) $\frac{255}{2}$ (b) $\frac{2}{255}$

(c) 0 (d) none of these

Ans. (a) $\frac{255}{2}$

Explanation: Given, $f(x) = 2x^3 - \left(\frac{1}{x}\right)^{1/2}$

Put, $x = 4$

$$f(4) = 2 \times 4^3 - \left(\frac{1}{4}\right)^{1/2}$$
$$f(4) = 2 \times 64 - \frac{1}{2}$$
$$f(4) = 128 - \frac{1}{2}$$
$$f(4) = \frac{256-1}{2}$$
$$f(4) = \frac{255}{2}$$

7. $f(x) = \frac{9}{5}x + 32$, the value of $f(-10)$ is:

(a) 15 (b) 14

(c) –15 (d) –14

Ans. (b) 14

Explanation: Given,

$$f(x) = \frac{9}{5}x + 32$$
$$\therefore\ f(-10) = \frac{9}{5}(-10) + 32$$
$$= 9 \times (-2) + 32$$
$$= -18 + 32$$
$$= 14$$

8. Let $f(x) = \begin{cases} 3 - 2x & x < 1 \\ 3x + 5 & 1 \leq x \leq 2, \\ 5x - 2 & x > 3 \end{cases}$

then $2f(0) + f(3)$ is:

(a) 17 (b) 19

(c) 24 (d) 31 [Diksha]

Ans. (b) 19

Explanation: Here,

$$f(0) = 3 - 2(0) = 3$$
$$f(3) = 5 \times 3 - 2 = 13$$

So,

$$2f(0) + f(3) = 2 \times 3 + 13$$
$$= 6 + 13$$
$$= 19$$

9. If $A = \{a, b, c\}$ $B = \{x, y\}$, then the number of functions that can be defined from A into B is:

(a) 12 (b) 8

(c) 6 (d) 3

Ans. *(b) 8*

Explanation: Here, $n(A) = 3 = m$

$n(B) = 2 = m$

We know that, the number of functions from

$A \to B$ is n^m

Therefore, the number of functions that can be defined from A into B is $= 2^3 = 8$.

10. Is the given relation is a function?

$R = \{(1, 3)\ (2, 4)\ (2, 3)\ (3, 4)\}$.

(a) Yes

(b) No

(c) Cannot say

(d) Insufficient information

Ans. *(b) No*

Explanation: Here, domain of the relation is repeated.

Hence, it is not a function.

11. If $[x]^2 - 5\ [x] + 6 = 0$, where [.] denotes the greatest integer function, then

(a) $x \in [3, 4]$ (b) $x \in (2, 3]$

(c) $x \in [2, 3]$ (d) $x \in [2, 4)$

[NCERT Exemplar]

Ans. *(c) $x \in [2, 3]$*

Explanation: We have,

$$[x]^2 - 5\ [x] + 6 = 0$$
$$\Rightarrow [x]^2 - 3\ [x] - 2\ [x] + 6 = 0$$
$$\Rightarrow [x]\ ([x] - 3) - 2\ ([x] - 3) = 0$$
$$\Rightarrow ([x] - 3)\ ([x] - 2) = 0$$
$$\Rightarrow [x] = 2, 3$$

So, $x \in [2, 3]$.

12. Domain of $\sqrt{7 - x^2}$ is:

(a) $[0, \sqrt{7}]$ (b) $[-\sqrt{7}, \sqrt{7})$

(c) $[\sqrt{7}, 0]$ (d) none

Ans. *(b) $[-\sqrt{7}, \sqrt{7})$*

Explanation: Here, $7 - x^2 \geq 0$

$$7 \geq x^2$$
$$\sqrt{7} \geq x > \sqrt{7}$$

Domain is $[-\sqrt{7}, \sqrt{7})$.

13. If $[x]^2 - 5[x] + 6 = 0$, where [.] denote the greatest integer function, then:

(a) $x \in [3, 4]$ (b) $x \in (2, 3]$

(c) $x \in [2, 3]$ (d) $x \in [2, 4]$

[Delhi Gov. QB 2022]

Ans. *(c) $x \in [2,3]$*

Explanation: We have,

$$[x]^2 - 5[x] + 6 = 0$$
$$\Rightarrow [x]^2 - 3[x] - 2[x] + 6 = 0$$
$$\Rightarrow [x]([x] - 3) - 2([x] - 3) = 0$$
$$\Rightarrow ([x] - 3)([x] - 2) = 0$$
$$\Rightarrow [x] = 2, 3$$

So $x \in [2, 3]$.

Assertion-Reason Questions

Direction: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

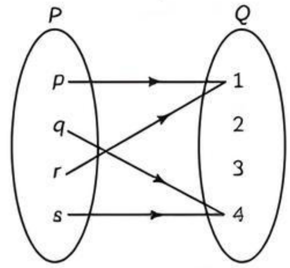
(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

14. Assertion (A): The following arrow diagram represents a function.



Reason (R): Let $f : R - \{2\} \to R$ be defined by $f(x) = \dfrac{x^2 - 4}{x - 2}$ and $g : R \to R$ be define by $g(x) = x + 2$. Then $f = g$.

Ans. *(c) (A) is true but (R) is false.*

Explanation: In the arrow diagram, each element of P has its unique image in Q.

Hence, the following arrow diagram represent a function.

We have, $f(x) = \dfrac{x^2 - 4}{x - 2}, x \neq 2$

$$\Rightarrow f(x) = \frac{(x-2)(x+2)}{x-2}, x \neq 2$$

$$\Rightarrow f(x) = x + 2, x \neq 2$$

But, $g(x) = x + 2$ for all $x \in R$

Then, $f(x)$ and $g(x)$ have different domains.
Domain of $f = R - \{2\}$ and domain of $g = R$

$\therefore \quad f \neq g$

15. **Assertion (A):** The range of function $f(x) = 2 - 3x$ is R.

Reason (R): The range of function $f(x) = x^2 + 2$ is $[2, \infty)$.

Ans *(d) (A) is false but (R) is true.*

Explanation: Given, $f(x) = 2 - 3x$

Let, $y = 2 - 3x$

$$x = \frac{2-y}{3} \text{ for } x > 0, y < 2$$

Hence, Range $= (-\infty, 2)$.

Now, $f(x) = x^2 + 2$

$y = x^2 + 2$

$\Rightarrow \quad x = \sqrt{y-2}$

Hence, Range $= [2, \infty)$

16. **Assertion (A):** The Domain of the Real function f defined by $f(x) = \sqrt{x-1}$ is $R - \{1\}$.

Reason (R): The Range of function $f(x) = \sqrt{x-1}$ is $[0, \infty)$.

Ans. *(d) (A) is false but (R) is true.*

Explanation: $f(x) = \sqrt{x-1}$

$x - 1 \geq 0, x \geq 1$

$D = [1, \infty)$

$y = \sqrt{x-1}$

$x \geq 1$

Range $= [0, \infty)$

17. **Assertion (A):** Let $A = \{1, 2, 3, 5\}$, $B = \{4, 6, 9\}$ and $f = \{(x, y) : |x - y| \text{ is odd}\}$. Then domain of f is $\{1, 2, 3, 5\}$

Reason (R): $|x|$ is always positive $\forall x \in R$

Ans. *(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).*

Explanation: Given

$R = \{(x, y) : |x - y| \text{ is odd}, x \in A, y \in B\}$

$R = \{(1, 4) (1, 6) (2, 9) (3, 4) (3, 6) (5, 4) (5, 6)\}$

Domain of $R = \{1, 2, 3, 5\}$

$|x|$ is always positive $\forall x \in R$.

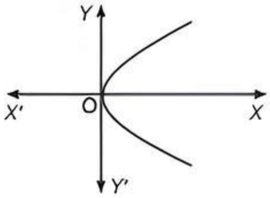
CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

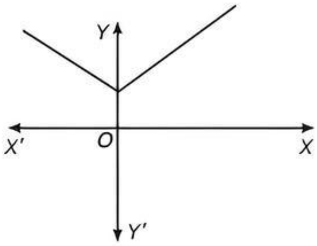
18. *Function as a Relation from a non-empty set A to a non-empty set B is said to be a function if every element of set A has one and only one image in set B. In other words, we can say that a function f is a relation from a non-empty set A to a non-empty set B such that the domain of f is A and no two distinct ordered pairs in f have the same first element or component. If f is a function from a set A to a set B, then we write f : A → B and it is read as f is a function from A to B or f maps A to B.*

(A) The given curve is a:



(a) function
(b) relation
(c) can't say anything
(d) data not sufficient

(B) The given curve is a:



(a) function
(b) relation
(c) can't say anything
(d) data not suffcient

(C) If $f(x) = x^2 + 2x + 3$, then among $f(1)$, $f(2)$ and $f(3)$, which one gives the maximum value.

(a) $f(1)$
(b) $f(2)$
(c) $f(3)$
(d) $f(1) = f(2) = f(3)$

(D) If $f(1 + x) = x^2 + 1$, then $f(2 - h)$ is:

(a) $h^2 - 2h - 1$
(b) $h^2 - 2h + 1$
(c) $h^2 - 2h + 2$
(d) $h^2 + 2h + 2$

(E) Assertion (A): The cartesian product $A \times A$ has 9 elements among which are found (–1, 0) and (0, 1). The set A and the remaining elements of $A \times A$ are (–1, –1), (–1, 1), (0, –1), (0, 0), (1, –1), (1, 0) and (1, 1).

Reason (R): If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

Ans. (A) *(b) relation*

Explanation: If we draw a vertical line, then it will intersect the curve at two points. It shows that a given curve is a relation.

(B) *(a) function*

Explanation: If we draw a vertical line, then it will intersect the curve at only one point. It shows that a given curve is a function.

(C) *(c) f(3)*

Explanation: $f(1) = 1 + 2 + 3 = 6$,

$f(2) = 4 + 4 + 3 = 11$

and $f(3) = 9 + 6 + 3 = 18$.

Here, 18 is the maximum value.

(D) *(c)* $h^2 - 2h + 2$

Explanation: We have, $f(1 + x) = x^2 + 1$ ...(i)

On substituting $x = (1 - h)$ in eq. (i), we get

$$f(1 + 1 - h) = (1 - h)^2 + 1$$
$$f(2 - h) = 1 + h^2 - 2h + 1$$
$$= h^2 - 2h + 2$$

(E) *(a) Both (A) and (R) are true and (R) is the correct explanation of (A).*

Explanation: We know that,

If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$

From the given,

$$n(A \times A) = 9$$
$$n(A) \times n(A) = 9,$$
$$n(A) = 3 \quad \text{...(i)}$$

The ordered pairs (–1, 0) and (0, 1) are two of the nine elements of $A \times A$.

Therefore, $A \times A = \{(a, a) : a \in A\}$

Hence, –1, 0, 1 are the elements of A ...(ii)

From (i) and (ii),

$$A = \{-1, 0, 1\}$$

The remaining elements of set $A \times A$ are (–1, –1), (–1, 1), (0, –1), (0, 0), (1, –1), (1, 0) and (1, 1).

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

19. **Let $f = \{(2, 2), (3, 4), (0, -2), (-1, -4)\}$ be a function from Z to Z defined by $f(x) = ax + b$, for some integers a, b. Determine a, b.**

Ans. Given, $f = \{(2, 2), (3, 4), (0, -2), (-1, -4)\}$ be a function from Z to Z defined by $f(x) = ax + b$.

We have,

$$f(0) = -2$$
$$\Rightarrow \quad b = -2 \quad \text{...(i)}$$
$$f(-1) = -4$$
$$\Rightarrow \quad -a + b = -4 \quad \text{...(ii)}$$

On solving the above equation, we get

$a = 2, b = -2$.

20. **If $f(x) = y = \dfrac{ax - b}{cx - a}$, then prove that $f(y) = x$.**

Ans. We have,

$$f(x) = y = \frac{ax - b}{cx - a} \quad \text{...(i)}$$

Now, we will replace x with y, we get

$$f(y) = \frac{a(y) - b}{c(y) - a}$$

From equation (i), we will substitute the value of y in above equation, we get

$$f(y) = \frac{a\left(\dfrac{ax - b}{cx - a}\right) - b}{c\left(\dfrac{ax - b}{cx - a}\right) - a}$$

$$= \frac{\dfrac{a(ax - b)}{(x - a)} - b}{\dfrac{c(ax - b)}{cx - a} - a}$$

$$= \frac{\dfrac{a(ax - b) - b(cx - a)}{cx - a}}{\dfrac{c(ax - b) - a(cx - a)}{cx - a}}$$

$$= \frac{a(ax - b) - b(cx - a)}{c(ax - b) - a(cx - a)}$$

$$= \frac{a^2x - ab - bcx + ab}{acx - bc - acx + a^2}$$

$$= \frac{a^2x - bcx}{a^2 - bc}$$

$$f(y) = \frac{x(a^2 - bc)}{-bc + a^2}$$

Cancelling the like terms, we get

$$f(y) = x$$

Hence, proved.

21. Find the domain and range of the real function $f(x) = \sqrt{x-2}$.

Ans. Given, $f(x) = \sqrt{x-2}$.

Since, $f(x)$ is defined for every $x \in R$ Such that

$x - 2 \geq 0$, i.e., $x \geq 2$.

The domain of the function is the value at which the function has a possible real value of range.

Hence, the domain of $f = [2, \infty)$.

Let $y = f(x)$

$\Rightarrow \quad y = \sqrt{x-2}$

$\Rightarrow \quad y^2 = x - 2$

$\Rightarrow \quad x = 2 + y^2$,

which is defined for every $y \in R$.

Also, $y = \sqrt{x-2} \geq 0$

Range = $[0, \infty)$.

22. If $f(x) = x^3$, find the value of $\frac{f(5) - f(1)}{5 - 1}$.

[Delhi Gov. QB 2022]

Ans. Given, $f(x) = x^3$

$$\frac{f(5) - f(1)}{5 - 1} = \frac{5^3 - 1^3}{5 - 1}$$

$$= \frac{125 - 1}{4}$$

$$= \frac{124}{4} = 32$$

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

23. Let f and g be real functions defined by $f(x) = 2x + 1$ and $g(x) = 4x - 7$

(A) For what real numbers x, $f(x) = g(x)$?

(B) For what real numbers x, $f(x) < g(x)$?

[NCERT Exemplar]

Ans. Given that:

$f(x) = 2x + 1$ and $g(x) = 4x - 7$

(A) For $f(x) = g(x)$, we get

$2x + 1 = 4x - 7$

$\Rightarrow \quad 2x - 4x = -7 - 1$

$\Rightarrow \quad -2x = -8$

$\Rightarrow \quad x = 4$

Hence, the required real number is 4.

(B) For $f(x) < g(x)$, we get

$2x + 1 < 4x - 7$

$\Rightarrow \quad 2x - 4x < -1 - 7$

$\Rightarrow \quad -2x < -8$

$\Rightarrow \quad 2x > 8$

$\therefore \quad x > 4$

Hence, the required real number is $x > 4$.

24. $A = \{1, 2, 3, 4, 5\}$, $S = \{(x, y) : x \in A, y \in A\}$, then find the ordered which satisfy the conditions given below.

(A) $x + y = 5$ (B) $x + y < 5$

Ans. We have,

$A = \{1, 2, 3, 4, 5\}$, $S = \{(x, y) : x \in A, y \in A\}$

(A) The set of ordered pairs satisfying $x + y = 5$ is, $\{(1, 4), (2, 3), (3, 2), (4, 1)\}$.

(B) The set of ordered pairs satisfying $x + y < 5$ is $\{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)\}$

25. Find the domain of each of the following functions:

(A) $f(x) = -|x - 2|$ (B) $f(x) = x|x|$.

Ans. (A) Given, $f(x) = -|x - 2|$.

Since, $f(x)$ is defined for every $x \in R$.

Domain $(f) = R$

Hence, the domain of $f = R$.

(B) Given $f(x) = x|x|$.

Since $f(x)$ is defined for every $x \in R$.

Hence, the domain of $f = R$.

26. Find the domain of the following functions:

(A) $f(x) = \frac{1}{\sqrt{x - [x]}}$ (B) $f(x) = \frac{1}{\sqrt{x + [x]}}$

Ans. (A) Given $f(x) = \frac{1}{\sqrt{x - [x]}}$

We have, $x - [x]$: Decimal part of x.

Since, $f(x)$ is defined for every $x \in R$ such that

Hence, the domain of $f = R - Z$

(B) We have, $f(x) = \frac{1}{\sqrt{x + [x]}}$

Now, $|x| = \begin{cases} x, & \text{when } x \geq 0 \\ -x, & \text{when } x < 0 \end{cases}$

$\Rightarrow \quad -x+|x| = \begin{cases} x+x, & \text{when } x \geq 0 \\ x-x, & \text{when } x < 0 \end{cases}$

$\Rightarrow \quad x+|x| = \begin{cases} 2x, & \text{when } x \geq 0 \\ 0, & \text{when } x < 0 \end{cases}$

$\Rightarrow \quad x+|x| > 0$, when $x > 0$

$\Rightarrow \quad f(x) = \dfrac{1}{\sqrt{x+[x]}}$

assumes real values only when, $x + |x| > 0$ and this happens only when, $x > 0$

$\therefore$ Domain of $(f) = (0, \infty)$.

27. **If A = {2, 4, 6, 9}, B = {4, 6, 18, 27, 54} and a relation R from A to B is defined by $R = \{(a, b) : a \in A, b \in B, a$ is a factor of b and $a < b\}$ then find in roster form. Also find its domain and range.** **[Delhi Gov. QB 2022]**

Ans. Given, A = {2, 4, 6, 9} and B = {4, 6, 18, 27, 54} and

$R = \{(a, b) : a \in A, b \in B, a$ is a factor of b and $a < b\}$

$\therefore$ Roster form, R = {(2, 4), (2, 6), (2, 18), (2, 54), (6, 18), (6, 54),(9, 18), (9, 27), (9, 54)}

Domain of R = {2, 6, 9}

Range of R = {4, 6, 18, 27, 54}

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

28. **Find the domain of $\dfrac{x^2+2x+7}{x^2-x-6}$.**

Ans. We have, $f(x) = \dfrac{x^2+2x+7}{x^2-x-6}$

Clearly, $f(x)$ is a rational function of x as $\dfrac{x^2+2x+7}{x^2-x-6}$ is a rational expression in x. We observe that $f(x)$ assumes real values for all x except for all those values of x for which $x^2 - x - 6 = 0$ i.e. $x = 3, -2$.

Hence, Domain $(f) = R - \{3, -2\}$.

29. **Find the domain of the function $f(x)$ defined by $f(x) = \sqrt{4-x} + \dfrac{1}{\sqrt{x^2-1}}$.**

Ans. Given, $f(x) = \sqrt{4-x} + \dfrac{1}{\sqrt{x^2-1}}$

Clearly, $f(x)$ is defined for all x satisfying for domain of the given function both

$4 - x \geq 0$ and $x^2 - 1 > 0$

$\Rightarrow \quad x - 4 \leq 0$ and $(x - 1)(x + 1) > 0$

$\Rightarrow \quad x \leq 4$ and $(x < -1$ or $x > 1)$

On combining both conditions and finding the combined other regions, we get,

$\Rightarrow x \in (-\infty, -1) \cup (1, 4]$.

Hence, Domain $(f) = (-\infty, -1) \cup (1, 4]$.

30. **Let f and g be two real functions defined by $f(x) = \dfrac{1}{x-6}$ and $g(x) = (x-6)^2$.**

Find the following:

(A) $f+g$ **(B) $f-g$**

(C) fg

Ans. We observe that, $f(x) = \dfrac{1}{x-6}$ is defined for all $x = 6$. So, domain $(f) = R - \{6\}$

Clearly, $g(x) = (x-6)^2$ is defined for all $x \in R$. So, domain $(g) = R$.

$\therefore$ Domain $(f) \cap$ Domain $(g) = R - \{6\}$.

(A) $f + g : R - \{6\} \to R$ is given by

$(f+g)(x) = f(x) + g(x)$

$= \dfrac{1}{x-6} + (x-6)^2$

$= \dfrac{(x-6)^3+1}{x-6}$

(B) $f - g : R - \{6\} \to R$ is defined as

$(f-g)(x) = f(x) - g(x)$

$= \dfrac{1}{x-6} - (x-6)^2$

$= \dfrac{1-(x-6)^3}{x-6}$

(C) $fg : R - \{6\} \to R$ is given by

$(fg)(x) = f(x)\, g(x)$

$= \dfrac{1}{x-6} \times (x-6)^2 = (x-6)$

31. **Find the range of each of the following functions:**

(A) $f(x) = |x - 6|$ **(B) $f(x) = 1 - |x - 4|$**

(C) $f(x) = \dfrac{|x-2|}{x-2}$

Ans. (A) We have $f(x) = |x - 6|$

Clearly, $f(x)$ is defined for all $x \in R$. Therefore, Domain $(f) = R$.

$\because \quad |x - 6| > 0$ for all $x \in R$

$\therefore \quad 0 \leq |x - 6| < \infty$ for all $x \in R$

$\Rightarrow \quad 0 \leq f(x) < \infty$ for all $x \in R$

$\Rightarrow$ $f(x) \in [0, \infty)$ for all $x \in R$

Hence, Range $(f) = [0, \infty)$.

(B) We have, $f(x) = 1 - |x - 4|$.

We observe that, $f(x)$ is defined for all $x \in R$.

Therefore, Domain $(f) = R$.

$\because$ $0 \le |x - 4| < \infty$ for all $x \in R$

$\therefore$ $-\infty < -|x - 4| \le 0$ for all $x \in R$

$\Rightarrow$ $-\infty < 1 - |x - 4| \le 1$ for all $x \in R$

$\Rightarrow$ $-\infty < f(x) \le 1$ for all $x \in R$

$\Rightarrow$ $f(x) \in (-\infty, 1]$

Hence, Range $(f) = (-\infty, 1]$

(C) We have $f(x) = \dfrac{|x-2|}{x-2}$

Clearly, $f(x)$ is defined for all $x \in R$ except at $x = 2$.

Therefore, Domain $(f) = R - \{2\}$

Now,

$$f(x) = \frac{|x-2|}{x-2}$$

Hence, Range $(f) = \{-1, 1\}$

32. Let $A = \{1, 2, 3, 4\}$, $B = \{1, 4, 9, 16, 25\}$ and R be a relation defined from A to B as,

$R = \{(x, y), x \in A, y \in B$ and $y = x^2)$

(A) Depict this relation using arrow diagram.

(B) Find domain of R.

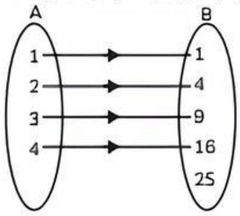
(C) Find range of R.

(D) Write co-domain of R

[Delhi Gov. QB 2022]

Ans. Given, $A = \{1, 2, 3, 4\}$ and $B = \{1, 4, 9, 16, 25\}$ and $R = \{(x, y) : x \in A, y \in B$ and $y = x^2)$

(A) Relation, $R = \{(1, 1), (2, 4), (3, 9), (4, 16)\}$



(B) Domain of $R = (1, 2, 3, 4)$

(C) Range of $R = (1, 4, 9, 16)$

(D) Codomain of $R = (1, 4, 9, 16, 25)$

33. If $R_1 = \{(x, y) \mid y = 2x + 7$ where $x \in R$ and $-5 \le x \le 5\}$ is a relation. Then find the domain and range. [NCERT Exemplar]

Ans. Given that: $R_1 = \{(x, y) \mid y = 2x + 7$ where $x \in R$ and $-5 \le x \le 5\}$ is a relation.

The domain of R_1 consists of all the first elements of all the ordered pairs of R_1.

Here domain is $-5 \le x \le 5 \Rightarrow \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$ and $y = 2x + 7$.

It is also given $y = 2x + 7$

Now $x \in [-5, 5]$

Multiply LHS and RHS by 2, we get

$$2x \in [-10, 10]$$

Adding LHS and RHS with 7 we get, $2x + 7 \in [-3, 17]$

So, the values of y for the corresponding given values of x are $\{-3, -1, 1, 3, 5, 7, 9, 11, 13, 15, 17\}$

Hence, the domain of $R_1 = [-5, 5]$ and range of $R_1 = [-3, 17]$.

34. Let $f = \left\{\left[x \cdot \dfrac{x^2}{1+x^2}\right] : x \in R\right\}$ be function from R to R. Determine range of f.

[Delhi Gov. SQP 2022]

Ans. Let $y = \dfrac{x^2}{1+x^2}$

$\Rightarrow$ $y + x^2 y = x^2$

$\Rightarrow$ $y = x^2(1 - y)$

$\Rightarrow$ $x^2 = \dfrac{y}{1-y}$

$\Rightarrow$ $x = \sqrt{\dfrac{y}{1-y}}$

Since, x is real

$\Rightarrow$ $\dfrac{y}{1-y} \ge 0$

$\Rightarrow$ $\dfrac{y(1-y)}{(1-y)^2} \ge 0$

$\Rightarrow$ $y(1 - y) \ge 0$ and $(1 - y^2) > 0$

$\Rightarrow$ $0 \le y \le 1$ and $-y > -1$

$\Rightarrow$ $0 \le y \le 1$ and $y < 1$

Hence, $0 \le y < 1$

Range of f is $[0, 1)$.

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

35. Find the range of each of the following functions:

(A) $f(x) = \dfrac{1}{\sqrt{x-3}}$ (B) $f(x) = \sqrt{36 - x^2}$

Ans. (A) We have, $f(x) = \dfrac{1}{\sqrt{x-3}}$

Clearly, $f(x)$ takes real values for all x satisfying $x - 3 > 0$

$\Rightarrow$ $x > 3$

$\Rightarrow \quad x \in (3, \infty)$.

$\therefore$ Domain (f) = $(3, \infty)$

For any $x > 3$ we have

$$x - 3 > 0$$

$$\Rightarrow \quad \sqrt{x-3} > 0$$

$$\Rightarrow \quad \frac{1}{\sqrt{x-3}} > 0$$

$$\Rightarrow \quad f(x) > 0$$

Thus, $f(x)$ takes all real values greater than zero. Hence, Range (f) = $(0, \infty)$.

(B) We have, $f(x) = \sqrt{36 - x^2}$

We observe that $f(x)$ is defined for all x satisfying

$$36 - x^2 \geq 0$$

$$\Rightarrow \quad x^2 - 36 \leq 0$$

$$\Rightarrow \quad (x - 6)(x + 6) \leq 0$$

$$\Rightarrow \quad -6 \leq x \leq 6$$

$$\Rightarrow \quad x \in [-6, 6].$$

$\therefore$ Domain (f) = [-6, 6].

Let $y = f(x)$. Then,

$$y = \sqrt{36 - x^2}$$

$$\Rightarrow \quad y^2 = 36 - x^2$$

$$\Rightarrow \quad x^2 = 36 - y^2$$

$$\Rightarrow \quad x = \sqrt{36 - y^2}$$

Clearly, x will take real values, if

$$36 - y^2 \geq 0$$

$$\Rightarrow \quad y^2 - 36 \leq 0$$

$$\Rightarrow \quad (y - 6)(y + 6) \leq 0$$

$$\Rightarrow \quad -6 \leq y \leq 6$$

$$\Rightarrow \quad y \in [-6, 6]$$

Also,

$y = \sqrt{36 - x^2} \geq 0$ for all $x \in 0$ [-6, 6].

Therefore, $y \in [0, 6]$ for all $x \in [-6, 6]$.

Hence, Range (f) = [0, 6]

36. If f and g are real functions defined by $f(x) = x^2 + 7$ and $g(x) = 3x + 5$, find each of the following:

(A) $f(3) + g(-5)$

(B) $f\left(\frac{1}{2}\right) \times g(14)$

(C) $f(-2) + g(-1)$

(D) $f(t) - f(-2)$

(E) $\frac{f(t) - f(5)}{t - 5}$ if $t \neq 5$ [NCERT Exemplar]

Ans. Given that: $f(x) = x^2 + 7$ and $g(x) = 3x + 5$

(A) $f(3) + g(-5) = [(3)^2 + 7] + [3(-5) + 5]$

$= (9 + 7) + (-15 + 5)$

$= 16 - 10 = 6$

Hence, $f(3) + g(-5) = 6$

(B) $f\left(\frac{1}{2}\right) \times g(14) = \left[\left(\frac{1}{2}\right)^2 + 7\right] \times [3 \times 14 + 5]$

$= \left(\frac{1}{4} + 7\right) \times (42 + 5)$

$= \frac{29}{4} \times 47$

$= \frac{1363}{4}$

Hence, $f\left(\frac{1}{2}\right) \times g(14) = \frac{1363}{4}$

(C) $f(-2) + g(-1) = [(-2)^2 + 7] + [3(-1) + 5]$

$= (4 + 7) + (-3 + 5)$

$= 11 + 2 = 13$

Hence, $f(-2) + g(-1) = 13$

(D) $f(t) - f(-2) = (t^2 + 7) - [(-2)^2 + 7]$

$= t^2 + 7 - 11$

$= t^2 - 4$

Hence, $f(t) - f(-2) = t^2 - 4$.

(E) $\frac{f(t) - f(5)}{t - 5}, (t \neq 5) = \frac{(t^2 + 7) - ((5)^2 + 7)}{t - 5}$

$= \frac{t^2 + 7 - 32}{t - 5}$

$= \frac{t^2 - 25}{t - 5} = t + 5$

Hence, $\frac{f(t) - f(5)}{t - 5}$ $t \neq 5 = t + 5$.

37. Find domain and range $f(x) = \frac{1}{2 - \sin 3x}$.

Ans. Let $f(x) = \frac{1}{(2 - \sin 3x)}$

Domain of $f(x)$ is $(-\infty, \infty)$

$$2 - \sin 3x = \frac{1}{y}$$

$$\Rightarrow \quad \sin 3x = 2 - \frac{1}{y}$$

$$\Rightarrow \quad \sin 3x = \frac{2y-1}{y}$$

$$\Rightarrow \quad 3x = \sin^{-1}\left(\frac{2y-1}{y}\right)$$

$$\Rightarrow \quad x = \frac{1}{3}\sin^{-1}\left(\frac{2y-1}{y}\right)$$

$$\left[\because y = \frac{1}{2-\sin 3x} \therefore y > 0 \text{ as } -1 \le \sin 3x \le 1\right]$$

For x to be real

$$-1 \le \frac{2y-1}{y} \le 1$$

$$\Rightarrow \quad -y \le 2y - 1 \le y \qquad [y > 0]$$

$$\Rightarrow \quad 2y - 1 \ge -y \text{ and } 2y - 1 \le y$$

$$\Rightarrow \quad y \ge \frac{1}{3}, y \le 1$$

$\therefore$ Range of $f(x) = \left[\frac{1}{3}, 1\right]$

38. Are the given relations functions? Give reason for your answer.

(A) $h = \{(4, 6), (3, 9), (-11, 6), (3, 11)\}$.

(B) $f = \{(x, x) \mid x$ is a real number$\}$.

(C) $g = \left\{\left(x, \frac{1}{x}\right) \middle| x \text{ is a positive integer}\right\}$.

(D) $s = \{(x, x^2) \mid x$ is a positive integer$\}$.

(E) $t = \{(x, 3) \mid x$ is a real number$\}$.

Ans. (A) We have, $h = \{(4, 6), (3, 9), (-11, 6), (3, 11)\}$.
Since, 3 has two images 9 and 11. So, it is not a function.

(B) We have, $f = \{(x, x) \mid x$ is a real number.
We observe that, every element in the domain has unique image. So, it is a function.

(C) We have, $g = \left\{\left(x, \frac{1}{x}\right) \middle| x \text{ is a positive integer}\right\}$

For every x, it is a positive integer and $\frac{1}{x}$ is unique and distinct.
Therefore, every element in the domain has a unique image.
So, it is a function.

(D) We have, $s = \{(x, x^2) \mid x$ is a positive integer$\}$
Since, the square of any positive integer is unique.
So, every element in the domain has a unique image.
Hence, it is a function.

(E) We have, $t = \{(x, 3) \mid x$ is a real number$\}$
Since, every element in the domain has the image 3.
So, it is a constant function.

39. Express the following function as set of ordered pairs and determine their range.
$f: X \to R$, $f(x) = x^3 + 1$, where $X = \{-1, 0, 3, 9, 7\}$

[NCERT Exemplar]

Ans. A function $f: X \to R$, $f(x) = x^3 + 1$
Where $X = \{-1, 0, 3, 9, 7\}$
Domain = f is a function such that the first elements of all the ordered pair belong to the set $X = \{-1, 0, 3, 9, 7\}$.
The second element of all the ordered pair are such that they satisfy the condition

$$f(x) = x^3 + 1$$

When $x = -1$

$$f(x) = x^3 + 1$$
$$f(-1) = (-1)^3 + 1$$
$$= -1 + 1$$

$\Rightarrow$ Ordered pair = $(-1, 0)$

When $x = 0$,

$$f(x) = x^3 + 1$$
$$f(0) = (0)^3 + 1$$
$$= 0 + 1 = 1$$

$\Rightarrow$ Ordered pair = $(0, 1)$

When $x = 3$,

$$f(x) = x^3 + 1$$
$$f(3) = (3)^3 + 1$$
$$= 27 + 1$$
$$= 28$$

$\Rightarrow$ Ordered pair = $(3, 28)$

When $x = 9$,

$$f(x) = x^3 + 1$$
$$f(9) = (9)^3 + 1$$
$$= 729 + 1$$
$$= 730$$

$\Rightarrow$ Ordered pair = $(9, 730)$

When $x = 7$

$$f(x) = x^3 + 1$$
$$f(7) = (7)^3 + 1$$
$$= 343 + 1$$
$$= 344$$

$\Rightarrow$ Ordered pair = $(7, 344)$

Therefore, the given function as a set of ordered pairs is
$f = \{(-1, 0), (0, 1), (3, 28), (7, 344), (9, 730)\}$
and Range of $f = \{0, 1, 28, 730, 344\}$